THE SPECTRAL ESTIMATES FOR THE NEUMANN-LAPLACE OPERATOR IN SPACE DOMAINS

VLADIMIR GOL'DSHTEIN

The classical result by Payne and Weinberger states that in convex domains the first non-trivial eigenvalue of the Neumann-Laplace operator satisfies to the following inequality:

$$\mu_1(\Omega) \geq \frac{\pi^2}{d(\Omega)^2},$$

where $d(\Omega)$ is a diameter of a convex domain Ω.

In [1] we obtained lower estimates of $\mu_1(\Omega)$ in terms of a hyperbolic (conformal) radius of Ω for domains $\Omega \subset \mathbb{R}^2$ for a large class of non-convex domains. For space domains the theory of conformal mappings is not relevant.

We prove discreteness of the spectrum of the Neumann-Laplacian in a large class of non-convex space domains. The lower estimates of the first non-trivial eigenvalue are obtained in terms of geometric characteristics of homeomorphisms that induce composition operators on homogeneous Sobolev spaces. The suggested method is based on Poincaré-Sobolev inequalities that are obtained with the help of the geometric theory of composition operators. A corresponding composition operators are induced by a generalizations of conformal homeomorphisms that are mappings of bounded 2-dilatation (weak 2-quasiconformal mappings).

(Joint work with Alexander Ukhlov)

References

[1] V. Gol'dshtein, A. Ukhlov, On the First Eigenvalues of Free Vibrating Membrane in Conformal Regular Domains, Arch. Ration. Mech. Anal. 221 (2016), 893–915.