Lions' formula for RKHSs of real harmonic functions on Lipschitz domains

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Abstract. Let $\Omega \subset \mathbb{R}^n$, be a bounded Lipschitz domain. The purpose of this paper is to establish a new representaion of the formula due to J.L-Lions for reproducing kernel Hilbert spaces $\mathcal{H}^s(\Omega)$ of real harmonic functions on the usual Sobolev space $H^s(\Omega)$ for $s \geq 0$.
To this end, we provide a functional characterization of $\mathcal{H}^s(\Omega)$ via some new families of positive self-adjoint operators, describe their trace data and discuss the values of s for which they are RKHSs. Also a construction of an orthonormal basis of $\mathcal{H}^s(\Omega)$ is established.

Key words: harmonic function, reproducing kernel, the trace operator, the embedding operator

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