Mini-courses in Mathematical Analysis 2023

Book of abstracts

Mini-courses

Eigenvalue problems on singularly perturbed domains

Veronica FELLI

Università degli Studi di Milano-Bicocca, Italy

This mini-course focuses on the behavior of eigenvalues of the Dirichlet Laplacian when a small set is removed from the domain. We will develop a perturbative theory, considering the capacity of the removed set as the perturbation parameter. Additionally, we will perform a blow-up analysis for capacitary potentials and eigenfunctions, enabling us to derive explicit asymptotic expansions in some cases.

Spectral geometry of tubes

David KREJČIŘÍK

Czech Technical University in Prague, Czech Republic

The goal of this lecture is to give an account on spectral properties of tubular neighbourhoods of Riemannian manifolds. We focus on physical realisations when the base manifold is a curve in the three-dimensional Euclidean space, intensively studied during the last three decades because of physical motivations in quantum waveguides.

Around Fuglede's tiling-spectrality conjecture

Nir LEV

Bar-Ilan University, Israel

A domain $\Omega \subset \mathbb{R}^d$ is said to be *spectral* if the space $L^2(\Omega)$ admits an orthogonal basis of exponential functions. In 1974, Fuglede made a fascinating conjecture that the spectral domains could be characterized geometrically as the domains which can tile the space by translations. While this conjecture was disproved for general sets, in a joint work with Máté Matolcsi we did prove that Fuglede's conjecture is true for convex domains in all dimensions. I will survey the classical methods in the area, and then discuss our technique that is based on a new "weak tiling" condition for spectrality.

Talks

Existence of normalized solutions to a class of $(2, q)$-Laplacian equations in $\mathbb{R}^N$ (online)

Laura BALDELLI

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A variety of contexts in life sciences, physics, biophysics, chemical reaction, and in other branches of pure and applied science cannot be investigated without taking into account nonlinear phenomena. Motivated by the fact that physicists are often interested in normalized solutions, we will discuss some recent results concerning existence and nonexistence of solutions $(\lambda, u) \in \mathbb{R} \times X$, with $X := H^1(\mathbb{R}^N) \cap D^{1,q}(\mathbb{R}^N)$ to the following $(2, q)$-Laplacian equation in the entire Euclidean space

$$-\Delta u - \Delta_q u = \lambda u + |u|^{p-2}u \quad \text{in} \quad \mathbb{R}^N \tag{1}$$

under the constraint

$$\int_{\mathbb{R}^N} |u|^2 dx = c^2,$$

where $\Delta_q u = \mathrm{div}(|\nabla u|^{q-2}\nabla u)$ is the q-Laplacian of u, $c > 0$, $1 < q < N$, $q \neq 2$, $2 < p < \min\{2^*, q^*\}$ and $s^* := sN/(N-s)$ is the critical Sobolev's exponent, for every $1 < s < N$.

As far as we know, this is the first result concerning the existence and multiplicity of normalized solutions to (1). Therefore, we provide a new perspective to the study of the $(2, q)$-Laplacian equation in the mass subcritical, critical, and supercritical cases, in the sense of the critical exponents $2(1 + 2/N)$, $q(1 + 2/N)$. In the mass subcritical case, we study a global minimization problem obtaining a ground-state solution. While, in the mass critical cases, we prove several nonexistence results by using asymptotic decays of particular externals.

It is known that there are many difficulties in treating the supercritical case because of the unboundedness of the energy functional on the constraint. In order to overcome this obstacle, a suitable Pohožaev constraint approach and a natural stretched functional are taken into account to derive, respectively, a ground state solution and infinitely many radial solutions. Indeed, the radial setting allows us to overtake some restrictions in order to obtain a wider result. Moreover, the different behavior of the quasilinear q-Laplacian depending on the cases $q < 2$ and $q > 2$ requires a more cumbersome and accurate analysis with respect to the classical Schrödinger equation where $q = 2$. Our proofs rely on variational tools.

These results are contained in a work written jointly with Tao Yang (Zhejiang Normal University, P.R. China).

References

L. Baldelli, T. Yang, Normalized solutions to a class of $(2, q)$-Laplacian equation, *arXiv:2212.14873*.

Fourier restriction estimates on the torus and certain $\Lambda(p)$−sets

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For any subset $E \subset \mathbb{Z}$, let $P_E(\mathbb{T})$ denote the subspace of trigonometric polynomials whose Fourier coefficients are supported in E. A set $E \subset \mathbb{Z}$ is said to be a $\Lambda(p)-$set, for some $p > 2$, if

$$\|f\|_{L^p(\mathbb{T})} \leq C\|f\|_{L^2(\mathbb{T})},$$

for all $f \in P_E(\mathbb{T})$. Examples of $\Lambda(p)-$sets are Sidon sets, and finite-order lacunary sets.

In this talk, we briefly review some basic facts about these thin sets, and we discuss some Fourier extension estimates on the torus for trigonometric polynomials with frequency support on such sets.

This is based on joint work with O. Bakas and J. Wright.

Stability of the Discrete Dipole Approximation

Martin COSTABEL

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Since almost 50 years, the Discrete Dipole Approximation (DDA) has been a very popular numerical method for the dielectric scattering problem of time-harmonic electromagnetic waves with diverse fields of applications such as astrophysics, meteorology, nano-materials, or molecular biology. It is a discretization method for strongly singular volume integral equations that is very simple to implement, fast and apparently reliable.

Until very recently, however, there was no mathematical analysis available, because the method does not fit into standard frameworks of numerical methods for integral equations. Recently, we have obtained some stability estimates that show that the problem is non-trivial. While there are wide ranges of the physical parameters where the DDA is stable, there are cases (corresponding to high contrast materials) where the volume integral equation is well posed, but the DDA is unstable. In particular, it does not provide a spectrally correct approximation.

In the talk, I will describe some analytical tools used in the stability proofs, such as Ewald's summation method applied to slowly converging Fourier series, the application to Jacobi's theta functions of the maximum principle for the heat equation, or the optical theorem.

Based on joint work with M. Dauge and Kh. Nedaiasl.

References

[1] M. Costabel, M. Dauge, Kh. Nedaiasl: Stability Analysis of a Simple Discretization Method for a Class of Strongly Singular Integral Equations. *arXiv : 2302.13159* (2023).

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Existence of minimizers for weighted L^p–Hardy inequalities

Ujjal DAS

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Let $p \in (1, \infty)$, $\alpha \in \mathbb{R}$, and $\Omega \subsetneq \mathbb{R}^N$ be a $C^{1,\gamma}$-domain with a compact boundary $\partial\Omega$, where $\gamma \in (0, 1]$. Denote by $\delta_\Omega(x)$ the distance of a point $x \in \Omega$ to $\partial\Omega$. Let $\widetilde{W}_0^{1,p;\alpha}(\Omega)$ be the completion of $C_c^\infty(\Omega)$ with respect to the norm

$$\|\varphi\|_{\widetilde{W}_0^{1,p;\alpha}(\Omega)} := \left(\||\nabla\varphi|\|^p_{L^p(\Omega,\delta_\Omega^{-\alpha})} + \|\varphi\|^p_{L^p(\Omega,\delta_\Omega^{-(\alpha+p)})} \right)^{1/p}.$$

We study the following two variational constants: the *weighted Hardy constant*

$$H_{\alpha,p}(\Omega) := \inf \left\{ \int_\Omega |\nabla\varphi|^p \delta_\Omega^{-\alpha} \mathrm{d}x \,\middle|\, \int_\Omega |\varphi|^p \delta_\Omega^{-(\alpha+p)} \mathrm{d}x = 1,\ \varphi \in \widetilde{W}_0^{1,p;\alpha}(\Omega) \right\},$$

and the *weighted Hardy constant at infinity*

$$\lambda^\infty_{\alpha,p} := \sup_{K \Subset \Omega} \inf_{W_C^{1,p}(\Omega\setminus K)} \left\{ \int_{\Omega\setminus K} |\nabla\varphi|^p \delta_\Omega^{-\alpha} \mathrm{d}x \,\middle|\, \int_{\Omega\setminus K} |\varphi|^p \delta_\Omega^{-(\alpha+p)} \mathrm{d}x = 1 \right\}.$$

We show that $H_{\alpha,p}(\Omega)$ is attained if and only if the spectral gap $\Gamma_{\alpha,p}(\Omega) := \lambda^\infty_{\alpha,p} - H_{\alpha,p}(\Omega)$ is strictly positive. Moreover, we obtain tight decay estimates for the corresponding minimizers.

Based on joint work with Yehuda Pinchover and Baptiste Devyver.

Resonances and bound states in the scattering by dielectric or negative devices

Monique DAUGE

Université de Rennes, France

Motivated by various devices in (micro and nano) photonics, we investigate some spectral features of transmission operators P in divergence form $-\text{div}\, a\nabla$ posed in the entire plane for piecewise constant coefficients a: While a is $\equiv 1$ outside a bounded "cavity" Ω, it takes a value $a_\text{c} \neq 1$ inside Ω. The case $a_\text{c} > 1$ relates to dielectric resonators, and the case $a_\text{c} < 0$ to metallic devices (or metamaterials). Besides its essential spectrum (the positive half line), P may have negative eigenvalues if a_c is negative. But the most interesting part in the spectral features of P is the set of its scattering resonances. We will briefly describe the framework of blackbox scattering in which our problem fits perfectly. Then we will present some recent results about resonances close to the imaginary axis (whispering gallery modes).

This talk is partly based on a common work with S. Balac and Z. Moitier (2021), and partly on a paper authored by C. Carvalho and Z. Moitier (2023).

Comparison and regularity results for some anisotropic problems

Giuseppina DI BLASIO

Università della Campania, Caserta, Italy

The aim of this seminar is to outline some results obtained in recent years concerning nonlinear anisotropic Dirichlet problems.

Relying upon anisotropic symmetrization we exhibit some comparison results for solutions of different class of anisotropic elliptic equations with Dirichlet boundary conditions.

This talk is based upon a joint project in collaboration with A. Alberico and F. Feo, and recent results obtained in collaboration with F. Feo and G. Zecca.

Neumann biharmonic eigenvalue problem on thin curved domains

Francesco FERRARESSO

Università degli Studi di Sassari, Italy

Domain perturbation theory for the eigenvalues of the Neumann Laplace operator on families of bounded, Lipschitz domains of $\mathbb{R}^N$ is nowadays a well-understood, yet complicated subject. In general we cannot expect spectral continuity of the eigenvalues when the domain is varying; Courant and Hilbert showed that small perturbations of the unit square can generate an additional zero eigenvalue.

For the Neumann biharmonic operator, the situation is more involved, mainly due to two additional hurdles: 1) Neumann boundary conditions for the biharmonic operator are very sensitive to the curvature of the boundary; 2) standard techniques, such as the separation of variables, are not available.

After a review of the main results and counterexamples, I will focus on a specific singular perturbation where spectral continuity fails: thin annuli in $\mathbb{R}^2$.

In particular, I will show that the limits of the eigenvalues of the Neumann biharmonic operator (a scalar operator) on thin annuli are eigenvalues of a system of ordinary differential equations with non-constant coefficients depending on the curvature of the boundary.

Based on a joint work with L. Provenzano.

Qualitative properties of a class of nonlinear elliptic equations on Riemannian manifolds

Alberto FERRERO

Università del Piemonte Orientale, Italy

We discuss about some qualitative properties for solutions of the Lane-Emden and the Gelfand equations on non compact complete Riemannian manifolds with a pole of symmetry satisfying suitable assumptions at infinity involving the sign of the sectional curvatures. In particular, we will mainly discuss the question of stability of solutions emphasizing the main differences with the results already known in the Euclidean case. Part of the talk will be also devoted to the applications of this kind of equations in astrophysics and fluid dynamics.

Uniform Fourier restriction for convex curves

Marco FRACCAROLI

Universität Bonn, Germany

The study of the Fourier restriction phenomena in $\mathbb{R}^n$ was initiated in the '70s by Fefferman, Stein and Tomas and has been an active research topic ever since. The operator $\mathcal{R}$ sending a Schwartz function f to $\widehat{f}_{|S}$, the restriction of its Fourier transform to a hypersurface S, is well defined. A Fourier restriction estimate is then an a priori estimate on $\mathcal{R}$ between $L^p(\mathbb{R}^n)$ and $L^q(S,\nu)$, allowing the extension of $\mathcal{R}$ to $L^p(\mathbb{R}^n)$. Such an estimate is achievable as long as ν is an appropriate measure encoding the curvature properties of S. The sharp range of exponents p and q in any arbitrary dimension $n \geqslant 3$ is the subject of a long standing open conjecture.

In the case of a $\mathcal{C}^2$ convex curve in $\mathbb{R}^2$ the problem is solved. Sjölin proved a uniform Fourier restriction result for such curves upon the choice of the affine arclength measure ν. More recently Müller, Ricci, and Wright addressed a different feature of the Fourier restriction phenomena, showing that for any $f \in L^p(\mathbb{R}^2)$ the regularized value of $\widehat{f}$ coincides with that of $\mathcal{R}f$ at ν-a.e. point of a $\mathcal{C}^2$ convex curve.

In this talk we extend the aforementioned results to the case of arbitrary convex curves in the plane, removing the $\mathcal{C}^2$ regularity condition on the curve. The main ingredient is the choice of a suitable extension of the affine arclength measure suggested by a construction of Oberlin.

Extension and embedding theorems for Campanato spaces on $C^{0,\gamma}$ domains

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Damiano GRECO

Swansea University, UK

We consider Campanato spaces with exponents λ, p on domains of class $C^{0,\gamma}$ in the N-dimensional Euclidean space endowed with a natural anisotropic metric depending on γ. We discuss several results including the appropriate Campanato's embedding theorem and we prove that functions of those spaces can be extended to the whole of the Euclidean space without deterioration of the exponents λ, p.

This is a joint work in collaboration with Prof. Pier Domenico Lamberti.

Compactness in Lipschitz spaces and around (online)

Piotr KASPRZAK

Adam Mickiewicz University in Poznań, Poland

Compactness is one of the most fundamental mathematical notions and because of that, after more than a century from its formal introduction, it still attracts great interest of researchers. It is so widespread that it seems nigh to impossible to even briefly mention all the theories where it plays a crucial role. Unfortunately, very often it is not so easy or straightforward to prove that certain sets (or operators) are (or are not) compact, and so there is a great need for finding easy-to-use compactness criteria. Among the most famous ones are the Ascoli–Arzèla and Fréchet–Kolmogorov theorems in the spaces of continuous and p-Lebesgue integrable functions, respectively. There are also less known relatives of those results for sequence spaces. Although all those criteria are formulated for different Banach spaces, they share some similarities. Namely, certain characteristics corresponding to the very nature of the considered space (connected with, for example, continuity for the space $C(X, \mathbb{R})$, integral for $L^p(X, \mathbb{R})$ or remainder of a series for l^p) must be small for all the elements of a given set so that this set and its description can be 'reduced' to a family of finitely many points. Therefore, a natural question arises. Is it possible to give a unified approach to various known compactness criteria which in its formulation would use only basic functional analytic and no space-specific notions, but at the same time it would be quite general (in the sense that it would work in any normed/Banach space) and easy-to-use in applications.

The aim of the talk will be to characterize (pre)compactness in the spaces of Lipschitz continuous mappings acting between a metric space (X, d) and a normed space E. Over the last years several attempts to find a such a compactness criterion have been made, to no avail unfortunately. Only very recently a full characterization has been obtain.

When thinking about possible approaches to proving a compactness criterion in a given normed space, two obvious methods come to mind: direct and indirect one. The former one is self explanatory. The latter one often consists of two steps: finding another – simpler – space (linearly) isomorphic with our given space and proving a compactness criterion in this simpler space. It turns out that in the case of Lipschitz mappings as the simpler space we can take the product of E and the space of bounded and continuous functions. The isomorphism can be constructed using the so-called de Leeuw's map which to each function $f\colon X \to E$ assigns the bounded continuous mapping $\Phi(f)$, defined for distinct $x, y \in X$ by the formula $\Phi(f)(x, y) = (f(x) - f(y))/d(x, y)$. There is only one 'little' detail that needs to be taken care of. Namely, the function $\Phi(f)$ is not defined on the whole product $X \times X$, but on its proper subset $(X \times X) \setminus \{(x, x) \mid x \in X\}$, which may not be compact even if X is. So, to prove a compactness criterion for Lipschitz continuous mappings, one cannot apply the classical Ascoli–Arzèla theorem. A new compactness result for E-valued continuous and bounded mappings defined on non-compact metric spaces is needed.

On flat bands in perturbed Archimedean tilings

Joachim KERNER

FernUniversität in Hagen, Germany

In this talk we discuss existence and robustness of flat bands in perturbed two-dimensional Archimedean tilings which are of increasing interest in applied sciences and physics. We focus on the so-called Kagome and Super-Kagome lattice and characterize, among other things, all possible edge weights that lead to flat bands. For a physically important subclass of edge weights,

we also provide a more detailed picture of the spectrum including a discussion of emerging band gaps.

This talk is based on recent joint work with M. Täufer (Hagen) and J. Wintermayr (Wuppertal).

The Stokes operator on manifolds with cylindrical ends

Mirela KOHR

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We study the Stokes operator on a manifold with cylindrical ends. To this purpose, we obtain useful Fredholm, regularity, and invertibility results. An important role is played by an adapted pseudodifferential calculus on manifolds with straight cylindrical ends which contains the inverses of its L^2-invertible, elliptic operators of nonnegative order. We also describe the construction of the corresponding Stokes layer potentials.

Joint work with Victor Nistor (Metz) and Wolfgang L. Wendland (Stuttgart).

Eigenvalue Problem Involving Non-self Adjoint Differential Operators for the Optical Bent Waveguides (online)

Rakesh KUMAR

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In modern times, integrated photonics devices have gained importance due to their highspeed signal processing capability. Optical waveguides are one of the basic integrated photonic devices used in several applications, such as communication, medical devices, sensors, and more. The prominent waveguides are straight and bent. These waveguides are investigated experimentally, numerically, and semi-analytically [1, 2].

The analytic study of the straight waveguides showed that the operator corresponding to its eigenvalue problem is self-adjoint. It has real eigenvalues, and eigenfunctions corresponding to distinct eigenvalues are orthogonal [3].

In this work analytical study for the bent waveguide is presented [4]. It is found that the operator involved in the governing eigenvalue problem is defined on the infinite domain, and it is non-self-adjoint [4]. There are not many general predictions about the properties of non-self-adjoint operators, such as the nature of eigenvalues, eigenfunctions, etc [5]. Further analysis of the bent waveguide problem showed that it has complex eigenvalues. In addition, eigenfunctions corresponding to distinct eigenvalues are orthogonal. This eigenvalue problem involves a bent radius as a parameter. When the parameter is large, this non-self-adjoint problem transforms into the self-adjoint problem, and complex eigenvalues change into real eigenvalues, i.e., the eigenvalue problem corresponds to bent waveguides transforming into straight waveguides. Moreover, the finiteness of the discrete spectrum is also studied based on the compactness of the operator involved in the bent waveguide problem.

Based on joint work with Dr. Kirankumar R. Hiremath (Indian Institute of Technology Jodhpur).

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Towards the optimality of the ball for the Rayleigh conjecture concerning the clamped plate

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The first eigenvalue of the Dirichlet Bilaplacian shall be interpreted physically as the principal frequency of a vibrating plate with clamped edge. In 1894, in the wake of his famous conjecture regarding vibrating membranes, Rayleigh conjectured that, upon prescribing the area, the vibrating clamped plate with least principal frequency is circular. In 1995 (101 years later), Nadirashvili solved the conjecture as stated by Rayleigh, that is in dimension 2. In 1995 still, Ashbaugh and Bengouria proved the analogous statement of the conjecture in dimension 3. Since then, the conjecture remains desperately open in dimension $d > 3$.

In this talk, we make our contribution and show that the conjecture holds true in any dimension as soon as the first eigenfunction of the optimal shape satisfies some special condition. More precisely, this condition requires the first eigenfunction to have no critical value except its global minimum and global maximum. Under this assumption, the Rayleigh conjecture is proved thanks to a refinement of Talenti's far-reaching comparison principle, which is obtained after a fine study of the geometry of the nodal domains of the very same eigenfunction.

Phaseless sampling of the short-time Fourier transform

Lukas LIEHR

University of Vienna, Austria

Let $V_g f : \mathbb{R}^2 \to \mathbb{C}$ denote the short-time Fourier transform (STFT) of a function f with respect to a window function g,

$$V_g f(x, \omega) = \int_{\mathbb{R}} f(t) \overline{g(t-x)} e^{-2\pi i \omega t} dt.$$

The well-established theory of time-frequency analysis shows that under mild assumptions on g, every $f \in L^2(\mathbb{R})$ is uniquely determined by the samples $V_g f(\Lambda)$ provided that $\Lambda \subset \mathbb{R}^2$ is a sufficiently dense lattice. Now consider a more stringent problem where only *phaseless samples* of $V_g f$ are available, that is, the sampling set is of the form

$$|V_g f(\Lambda)| = \{|V_g f(\lambda)| : \lambda \in \Lambda\}.$$

The reconstruction of f from phaseless STFT samples is known as the *STFT phase retrieval problem*. Clearly, if $h = \tau f$ for some $\tau \in \mathbb{C}$ with $|\tau| = 1$ then $|V_g f(\Lambda)| = |V_g h(\Lambda)|$, implying that a unique reconstruction is only possible up to the ambiguity of a multiplicative phase factor. Under which assumptions on g and Λ is a function f uniquely determined (up to a phase factor) by $|V_g f(\Lambda)|$? I will present a series of recent results on this problem. It turns out, that the phaseless sampling problem differs from ordinary sampling in a rather fundamental way: if $\Lambda = A\mathbb{Z}^2$, $A \in \mathbb{R}^{2\times 2}$, is a lattice then uniqueness for functions $f \in L^2(\mathbb{R})$ is unachievable, independent of the choice of the window function and the density of the lattice. On the other hand, uniqueness from phaseless samples located on a lattice is possible in certain proper subspaces of $L^2(\mathbb{R})$, such as compactly supported functions and shift-invariant spaces. Finally, we highlight that a multi-window regime makes the problem unique without restrictions to subspaces: sampling on lattices with respect to 4 window functions instead of a single one implies uniqueness.

Layer potentials on manifolds with cylindrical ends: the Laplace operator

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Victor NISTOR

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We study the method of Layer Potentials on manifolds with cylindrical ends. This includes domains in $\mathbb{R}^n$ with outlets at infinity. One of the main difficulties is the characterization of the Fredholm properties of the resulting integral operators, which requires information on the behavior at infinity. We apply our results to the study of the Laplacian. Applications for the Stokes system and further results on the Laplacian will be discussed in the subsequent talk by Mirela Kohr.

Joint work with Marius Mitrea and Mirela Kohr.

Hypercomplex monogenic functions in boundary value problems for biharmonic functions

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Sergiy PLAKSA

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We consider a commutative algebra $\mathbb{B}$ over the field of complex numbers with a basis $\{e_1, e_2\}$ satisfying the conditions $(e_1^2+e_2^2)^2 = 0$, $e_1^2+e_2^2 \neq 0$. Let D be a simply connected domain in the Cartesian plane xOy and $D_\zeta = \{xe_1+ye_2 : (x, y) \in D\}$. Every monogenic function having the classic derivative in D_ζ is expanded in the form

$$\Phi(xe_1 + ye_2) = U_1(x, y)\, e_1 + U_2(x, y)\, ie_1 + U_3(x, y)\, e_2 + U_4(x, y)\, ie_2$$

so that components U_j are biharmonic functions in D, i.e., $\Delta^2 U_j(x, y) = 0$ for $j = 1, 2, 3, 4$.

We consider a Schwartz-type boundary value problem: to find a function $\Phi\colon D_\zeta \longrightarrow \mathbb{B}$ which is monogenic in a domain D_ζ when limiting values of components U_1, U_3 are given on the boundary ∂D_ζ:

$$U_1(x, y) = u_1(\zeta)\,, \quad U_3(x, y) = u_3(\zeta) \qquad \forall\, \zeta = xe_1 + ye_2 \in \partial D_\zeta.$$

This problem is associated with the following problem: to find a biharmonic function $V(x, y)$ in D when boundary values of its partial derivatives $\partial V/\partial x$, $\partial V/\partial y$ are given on the boundary ∂D. The problem is also associated with the *principal biharmonic problem*: to find a biharmonic function $V(x, y)$ in D, which is continuously extended together with partial derivatives of the first order up to the boundary ∂D, when its values and values of its outward normal derivative are given on ∂D.

Using a hypercomplex analog of the Cauchy type integral, we reduce the mentioned Schwartz-type boundary value problem to a system of integral equations.

A piecewise continuous biharmonic problem in an angle and corresponding to it a Schwartz-type boundary value problem for monogenic functions in a commutative biharmonic algebra are also considered. These problems are reduced to a system of integral equations on the positive semiaxis. It is proved that on any segment of this semiaxis the set of solutions of the system coincides with the set of solutions of a certain system of Fredholm integral equations.

This is joint work with Dr. Serhii Gryshchuk.

Resolvent convergence in varying spaces

Olaf POST

Universität Trier, Germany

In this talk, I present some recent results on generalised norm resolvent convergence: Weidmann proposed such a concept by embedding everything in a common Hilbert space and consider convergence there. Another concept is to use so-called identification operators close to unitary operators. I will also comment on some practical applications.

This is a joint work with Sebastian Zimmer (Uni Trier).

Bounds for the magnetic Neumann eigenvalues in the plane

Luigi PROVENZANO

Sapienza Università di Roma, Italy

We consider the eigenvalues of the magnetic Laplacian on a bounded domain Ω of $\mathbb{R}^2$ with uniform magnetic field $\beta > 0$ and magnetic Neumann boundary conditions. We find upper and lower bounds for the ground state energy λ_1 and we provide semiclassical estimates in the spirit of Kröger for the first Riesz mean of the eigenvalues. We also discuss upper bounds for the first eigenvalue for non-constant magnetic fields $\beta = \beta(x)$ on a simply connected domain in a Riemannian surface.

Based on joint works with Bruno Colbois (Université de Neuchâtel), Alessandro Savo (Sapienza Università di Roma) and Corentin Léna (Università di Padova).

Existence results for elliptic systems on bounded and unbounded domains

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John VILLAVERT

University of Texas, Rio Grande Valley, USA

In this talk, we shall focus on the non-negative solutions to a broad family of elliptic problems in the setting of the whole space or in bounded star-shaped domains. The type of equations and systems within this family include classical ones arising in finding the best constant in functional inequalities and curvature problems from conformal geometry. The model example is perhaps the scalar equation

$$\operatorname{div}(a(x)Du) + f(x,u) = 0, \ u \geq 0, \ \text{in } \Omega,$$

where we place suitable conditions on the weight $a(x)$, the nonlinearity f and the domain $\Omega \subset \mathbb{R}^N$. In particular, under certain growth conditions on the source nonlinearities and geometric assumptions on the domain, we can establish various existence and non-existence results, including Liouville-type theorems. We will describe these main results and outline the techniques for generating the results, which center around moving plane methods and degree theoretic shooting methods.

Permittivity perturbations for time-harmonic Maxwell's equations

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Michele ZACCARON

Czech Technical University in Prague, Czech Republic

We will discuss a curlcurl-type problem arising from time-harmonic Maxwell's equation in a perfectly conducting cavity of $\mathbb{R}^3$. The system in its most general form reads as follows

$$\begin{cases} \varepsilon^{-1}\operatorname{curl}\mu^{-1}\operatorname{curl} u = \lambda u, & \text{in } \Omega, \\ u \times \nu = 0, & \text{on } \partial\Omega, \end{cases}$$

with Ω a bounded domain of $\mathbb{R}^3$ and ν denoting its outer unit normal. Here ε and μ are the electric permittivity and magnetic permeability respectively, and they describe the electromagnetic properties of the medium filling Ω. We will present some results regarding the behaviour of the spectrum of the eigenproblem with respect to perturbations of the electric permittivity tensor ε, which in general represented by a 3×3 symmetrix matrix valued function. We will also quickly address the problem of shape optimization, with a focus on the case of product domains of the type $\omega \times I$, where ω is a bounded domain of $\mathbb{R}^2$ and I a bounded interval of $\mathbb{R}$.