Mini-courses in Mathematical Analysis
Padova 2024

Topics of the lectures and talks

Mini-courses

When Additive Combinatorics meets Ergodic Theory and Fourier Analysis

Mariusz MIREK

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This will be a series of lectures about recent progress on norm and pointwise convergence problems for classical and multiple ergodic averages along polynomial orbits. A celebrated theorem of Szemeredi asserts that every subset of integers with nonvanishing upper Banach density contains arbitrarily long arithmetic progressions. We will discuss the significance of using ergodic theory and Fourier analysis in solving this problem. We will also explain how this problem led to the conjecture of Furstenberg-Bergelson-Leibman, which is a major open problem in pointwise ergodic theory. Relations with number theory and additive combinatorics will be also discussed.

Analysis on manifolds and applications to layer potentials

Victor NISTOR

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A very old and quite successful method to solve boundary value problems is the method of layer potentials. This method reduces the study of a partial differential equation (formulated on the interior of our domain U) to the study of an integral operator equation formulated on the boundary ∂U of our domain. This boundary will be (except some very special circumstances) a non-trivial manifold. We are thus led to study integral and differential equations on manifolds. The first three lectures will introduce some basic techniques for the analysis on manifolds. In the last two lectures, we will discuss applications to the method of layer potentials A very old and quite successful method to solve boundary value problems is the method of layer potentials. This method reduces the study of a partial differential equation (formulated on the interior of our domain U) to the study of an integral operator equation formulated on the boundary ∂U of our domain. This boundary will be (except some very special circumstances) a non-trivial manifold. We are thus led to study integral and differential equations on manifolds. The first three lectures will introduce some basic techniques for the analysis on manifolds. In the last two lectures, we will discuss applications to the method of layer potentials (including some joint results with Mirela Kohr and Wolfgang Wendland).

Min-max principle as a tool for spectral asymptotics

Konstantin PANKRASHKIN

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The min-max principle for the eigenvalues of a semibounded self-adjoint operator is a classical result of the spectral theory, but most graduate courses only contain a very limited number of illustrations of how it can be used. We would like to discuss some less obvious applications in the spectral analysis of differential operators containing small or large parameters. In particular, we show how it can be efficiently used to estimate the decay of eigenfunctions and truncation errors in unbounded domains and to study the eigenvalues of thin domains.

Some classic results in shape optimization

Cristina TROMBETTI

Università di Napoli "Federico II", Italy

In these lessons, the techniques of symmetrization will be introduced and their applications to shape optimization problems will be examined.

Talks

A friendly introduction to nonlinear spectral theory

Jurgen APPELL

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In view of the utmost importance of spectral theory for linear operators, it is not surprising that various attempts have been made to define and study spectra also for some classes of nonlinear operators. The purpose of the talk is to discuss some of these spectra, to analyze their properties, and to compare their advantages and drawbacks, with a particular emphasis on examples and counterexamples.

Keywords: nonlinear spectral theory, properties and applications.

Some bounds for principal frequencies in general domains

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The inradius of an open set is a geometric quantity naturally linked to its sharp Poincaré-Sobolev embedding constants. In general, providing twosided estimates for these quantities in terms of the inradius is not always possible, unless some topological or capacitary assumptions comes into play. More precisely, for planar open sets with assigned topology we obtained an extension of a result due to Osserman and Taylor in the 70's. As a byproduct, for these classes of open sets, we derived a reverse Cheeger's inequality. We cover the superconformal case in any dimension and discuss the asymptotic optimality of the constants involved in the estimates, as well.

Based on joint work with Lorenzo Brasco.

Keywords: principal frequencies, inradius, capacity.

Endpoint estimates for higher-order Marcinkiewicz multipliers

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Marcinkiewicz multipliers on the real line are bounded functions of uniformly bounded variation on each Littlewood-Paley dyadic interval. The corresponding multiplier operators are well known to be bounded on $L^p(\mathbb{R})$ for all $1 < p < \infty$. Optimal weak-type endpoint estimates for these operators have been studied by Tao and Wright who proved that they map locally $L \log^{1/2} L$ to weak L^1. In this talk, we consider higher-order Marcinkiewicz multipliers, that is multipliers of uniformly bounded variation on each interval arising from a higher-order lacunary partition of the real line. We present optimal weak-type endpoint estimates for the corresponding multiplier operators. These are established as a consequence of a more general endpoint result for a higher-order variant of a class of multipliers introduced by Coifman, Rubio de Francia, and Semmes and further studied by Tao and Wright.

Based on joint work with Odysseas Bakas, Ioannis Parissis, and Marco Vitturi.

Keywords: Marcinkiewicz multipliers, endpoint estimates, higher-order lacunary sets.

On the eigenvalues of the Reissner-Mindlin system in the vanishing thickness limit

Francesco FERRARESSO

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The Reissner-Mindlin (RM) system is widely used in applications to model the vibrations of an elastic plate of thickness $t > 0$. As $t \to 0^+$, it is common knowledge that the eigenvalues of the RM system converge to the eigenvalues of the Kirchhoff-Love model (that is, of the bilaplacian operator). However, the available proofs of this fact usually depend on both the dimension of the ambient space and on the boundary conditions. I will give an alternative proof based on the Stummel-Vainnikko compact convergence theory, which works in any dimension and with any of the physically-relevant boundary conditions. As a consequence of the abstract method of proof, the convergence in operator norm of some suitably bordered resolvent operators is achieved.

Based on joint work with D. Buoso (Piemonte Orientale).

Sharp second order regularity for widely degenerate elliptic equations

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We present some regularity results for local weak solutions to widely degenerate elliptic equations of the form

$$-\mathrm{div}\left((|Du|-1)_+^{p-1}\frac{Du}{|Du|}\right)=f \quad \text{in } \Omega, \tag{1}$$

where $p>1$ and Ω is an open subset of $\mathbb{R}^n$, with $n\geq 2$. Assuming either f belongs to the Besov space $B^{(p-2)/p}_{(p-1)/p,1,loc}(\Omega)$, if $p>2$, or to the Lebesgue space $L^{\frac{np}{n(p-1)+2-p}}_{loc}(\Omega)$, if $1<p\leq 2$, we show that $\mathcal{G}_{\alpha,p}\left((|Du|-1)_+\right)\in W^{1,2}_{loc}(\Omega)$, for every $\alpha\geq\frac{p+1}{2(p-1)}$, where we set

$$\mathcal{G}_{\alpha,p}(t):=\int_0^t\frac{s^{\frac{p-1+2\alpha}{2}}}{(s+1)^{\frac{1+2\alpha}{2}}}\,ds \qquad \text{for } t\geq 0.$$

The strategy is based on an integration by parts in fractional sense, together with a generalization to Besov spaces of the Nečas' negative norm Theorem, which states that taking a fractional derivative of negative order of f_{x_j} gives a fractional derivative of positive order. Here, the main novelty is that we prove a higher differentiability result for a nonlinear function of the gradient of weak solutions to (1) under sharp conditions on the right-hand side. This allows us to establish the higher integrability of Du under the same minimal requirements on the datum f.

Based on joint work with P. Ambrosio and A. Passarelli di Napoli.

Keywords: Degenerate elliptic equations, singular elliptic equations, sharp second-order regularity.

Boundary value problems for the Stokes system on manifolds with straight cylindrical ends

Mirela KOHR

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We study boundary value problems for the Stokes system on a manifold with straight cylindrical ends. We use a layer potential approach based on Fredholm, regularity, and invertibility results. An adapted pseudodifferential calculus on manifolds with straight cylindrical ends provides the L^2-invertibility of the Stokes operator.

Joint work with Victor Nistor (Metz) and Wolfgang L. Wendland (Stuttgart).

Polynomial ergodic theorems for continuous flows

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We will present recent results concerning convergence in norm and pointwise almost everywhere for certain multiparameter polynomial ergodic averages for continuous flows. We will pay special attention to quantitative aspects of pointwise convergence phenomena from the point of view of uniform oscillation estimates. We will also discuss connections with a multiparameter variant of the Bellow-Furstenberg problem.

Based on joint work with Dariusz Kosz, Mariusz Mirek and Paweł Plewa.

Keywords: pointwise convergence, norm convergence, Radon operator.

A general integral identity with applications to a reverse Serrin problem

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The talk aims to present a new general differential identity and an associated integral identity, which entails a pair of solutions of the Poisson equation with constant source term. This generalizes a formula that R. Magnanini and G. Poggesi previously proved and used to obtain quantitative estimates of spherical symmetry for the Serrin overdetermined boundary value problem. As a first application of this new general differential identity, we prove a quantitative symmetry result for the "*reverse Serrin problem*", which we will introduce. In passing, we obtain a rigidity result for solutions of the aforementioned Poisson equation subject to a constant Neumann condition.

Based on joint work with R. Magnanini and G. Poggesi.

Keywords: Serrin overdetermined problem, integral identities, quantitative estimates.

On the continuous extension of the logarithmic double layer potential to the Ahlfors-regular boundary of a domain

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The classical theory of the logarithmic double layer potential is developed in the case where the boundary is a Lyapunov curve or a Radon curve of bounded rotation. J. Král (1964) established a necessary and sufficient condition for the boundary, under which the logarithmic double layer potential is continuously extended from a domain to the boundary for all continuous densities of potential.

We consider the class of Ahlfors-regular curves includes as a subclass the curves from the mentioned Král's result (as well as the Lyapunov curves and the Radon curves).

For the logarithmic double layer potential, a necessary and sufficient condition for the continuous extension to the Ahlfors-regular boundary is established. Sufficient conditions involving subclasses of Ahlfors-regular curves are also considered. Illustrative examples are presented.

Keywords: logarithmic double layer potential, Cauchy-type integral, Ahlfors-regular curve.

Courant's nodal domain theorem does not hold for Dirichlet-to-Neumann eigenfunctions

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The classical Courant's nodal domain theorem states that the n-th eigenfunction of the Laplacian on a compact manifold has at most n nodal domains. The same result holds for Steklov eigenfunctions on a compact manifold with boundary. The classical argument of the proof, however, does not apply to Dirichlet-to-Neumann eigenfunctions, which are the traces of Steklov eigenfunctions on the boundary. We disprove the conjectured validity of Courant's theorem for D-t-N eigenfunctions. Namely, given a smooth manifold M, and integers K, N, we built a Riemannian metric on M for which the n-th D-t-N eigenfunction has at least K nodal domains for all $n = 1, ..., N$.

Based on joint work with Angela Pistoia (Sapienza Università di Roma) and Alberto Enciso (ICMAT Madrid).

Keywords: Courant's nodal domain theorem, Dirichlet-to-Neumann problem, eigenfunctions.

On the Riemann problem in the logarithmic case

Sergei ROGOSIN and Maryna DUBATOVSKAYA

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The talk is devoted to an extended application of the logarithmization method to the study of solvability of the Riemann problem. Here we deal with solution of the Riemann problem for two functions and use the logarithmization method to the product of 2×2 nonsingular matrices. It is proved that this method is applicable to monodromy matrices with any Jordan normal form. Using this fact we solve the Riemann problem in the so called logarithmic cases.

Based on joint work with Ludmila Khvoshchinskaya and Maryna Dubatovskaya.

Keywords: Riemann problem, monodromy, Jordan normal form, logarithmization method.

Sharp constants in L^p-Hardy and Rellich inequalities

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In this talk I will present new L^p-versions of some Hardy and Hardy-Rellich inequalities, focusing on perturbations of the Laplacian by a singular potential given in multipolar form and powers of distance to the boundary, respectively. Also, I will talk about the fundamental solutions of the Schrödinger operator perturbed by a Hardy bipolar potential and the removable and isolated singularities of such operator.

Based on joint work with Cristian Cazacu.

Keywords: sharp constants, inequalities, singular.

The asymptotic behaviour of the solutions to widely degenerate p-Laplace equations

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We present some regularity results for solutions to the Dirichlet problems

$$\begin{cases} -\mathrm{div}\left((\nabla u_p - 1)_+^{p-1}\,\frac{\nabla u_p}{\nabla u_p}\right) = f & \text{in } B_R, \\ u_p = 0 & \text{on } \partial B_R, \end{cases}$$

where $p > 1$ and B_R is the open ball centered at the origin with radius $R > 0$ Through a well-known result by Talenti, we explicitly express the gradient of the solution up outside the ball with a radius of 1, if the datum f is a non-negative radially decreasing function. This allows us to analyze the behaviour of up as $p \to 1^+$ and to establish some higher integrability results and Lipschitz continuity property, assuming the datum f in a suitable Lorentz space.

Keywords: Asymptotic behaviour, widely degenerate, higher regularity.

Oscillation estimates for Carleson operator on radial functions

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In this talk, we will discuss estimates for the oscillation seminorm of the multidimensional Carleson operator acting on radial functions from weighted $L^p(\mathbb{R}^d, |x|^\beta)$ space. In this work, we use recent observations that the oscillation seminorm works well with projection operators. Then we reduce the problem to the transference principle between Hankel transforms $\mathcal{H}_\alpha$ of different orders, which is similar in spirit to Rubio de Francia's transference of radial multipliers.

Keywords: Carleson operator, Hankel transform, oscillation seminorm.

The von Neumann approach for positive symmetric systems

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Friedrichs (1958) introduced the concept of symmetric positive systems also known as Friedrichs systems, in order to treat the differential equations that change their type (the Tricomi equation). The class of Friedrichs systems encompasses a wide variety of equations. A new interest arose in this area after the introduction of operator-theoretic approach (abstract Friedrichs operators (2007)).

In this talk, we present a connection of abstract Friedrichs operators with symmetric and skew-symmetric operators. One immediate consequence of this approach is a generalised von-Neumann type decomposition for symmetric operators. Briefly, we also present the connection to semigroups.

Based on joint work with Marko Erceg.

Keywords: symmetric positive first-order system of partial differential equations, Linear symmetric and selfadjoint operators (unbounded), universal parametrisation of extensions.