Mini-courses in Mathematical Analysis

Padova 2026

Lectures and talks

Lectures

Ergodic Theorems

Tatjana (Tanja) Eisner

Institute of Mathematics, University of Leipzig, Germany

The purpose of this lecture course is to briefly introduce the beautiful and important subject of ergodic theorems, both classical and modern.

Spectral Geometry of Differential forms

Alessandro Savo

Dipartimento di Scienze di Base e Applicate per l'Ingegneria della Sapienza Università di Roma, Italy

The main topics of the mini-course are listed below.

1. The Hodge Laplacian acting on differential forms on a Riemannian manifold. Harmonic forms and cohomology: the Hodge-de Rham theorem. Bochner formula and applications to eigenvalue estimates. When there is a boundary: Reilly formula.

2. The Hodge Laplacian on convex Euclidean domains: main estimates in terms of the John ellipsoid. Example: the Maxwell operator.

3. Dirichlet-to-Neumann operator on differential forms, applications.

4. The Hodge Laplacian on minimal hypersurfaces of the sphere: applications to index estimates in terms of the first Betti number.

5. The Hodge Laplacian on negatively curved domains, Mc Kean inequality for differential forms in hyperbolic space.

Prerequisites: basic knowledge of analysis and Riemannian manifolds, all relevant notions will be recalled, and heavy technical details will be avoided. The main scope is to discuss the geometric relevance of eigenvalues for differential forms.

Talks

Lavrentiev Phenomenon and integral representation of the lower semicontinuous envelope for functionals with non convex and non continuous Lagrangians

Tommaso Bertin

Impan (Varsavia), Polonia

In this seminar, we investigate classical integral functionals in the Calculus of Variations and study conditions on the Lagrangian that guarantee the absence of the Lavrentiev Phenomenon. In particular, we focus on Lagrangians that are non-convex, possibly highly discontinuous, and unbounded with respect to the gradient variable.

Our first step to prove the non-occurrence of the Lavrentiev Phenomenon is to prove the representation of the lower semicontinuous envelope of a functional defined in $W^{1,\infty}(\Omega)$ as an integral functional whose Lagrangian is given by the bipolar of the original one.

Specifically, we adapt a technique presented in the classical book by Ekeland and Temam for the bounded case, and a refinement of a method due to Cellina , for the unbounded case. The integral representation also allows us to apply recent results by Bousquet, Mariconda and Treu to the non-convex setting.

We establish the integral representation of the lower semicontinuous envelope and the corresponding absence of the Lavrentiev Phenomenon under very weak assumptions in the autonomous case and under suitable anti-jump conditions on the spatial variable in the non-autonomous case. Furthermore we prove the strong convergence of the approximating sequence for minimizers or under specific growth assumptions on the Lagrangian.

These results provide new insights into the interplay between regularity assumptions, relaxation methods, and the avoidance of the Lavrentiev Phenomenon, thereby extending recent advances to a broader class of variational problems.

This is a joint project with with G. Treu (Università di Padova) and P. Huguet (université de Toulouse III Paul Sabatier)

Keywords: Lavrentiev Phenomenon, relaxation, lower semicontinuous envelope.

Discrepancy for non-smooth convex bodies in high dimensions

Roberto Bramati

Department of Management, Information and Production Engineering, University of Bergamo, Italy

The discrepancy of a distribution of N points in the torus $\mathbb{T}^d$ with respect to a given family of test sets measures how far the points are from being uniformly distributed over that family. When the family consists of all translates of a fixed set, one can consider the L^2-average of the discrepancy over translations and use Fourier analytical methods to understand its size. Sharp lower bounds for such L^2 discrepancy in terms of N are known for wide classes of sets in $\mathbb{T}^2$, but much less is known in higher dimensions. In this talk, I will report on recent progress in this direction, focusing on a family of test sets with "cylindrical" symmetry that can be defined in any dimension. They are particularly interesting because they exhibit geometric features known to play a key role in discrepancy theory: flat regions, curved regions, and corners.

Based on joint work with Luca Brandolini and Alessandro Monguzzi.

Keywords: discrepancy theory, Fourier analysis, irregularities of distribution.

Lower bounds for the Hodge Laplacian spectrum on non-convex domains

---

Tirumala Venkata Chakradhar

School of Mathematics, University of Bristol, United Kingdom

The Hodge Laplacian, which generalises the Laplace-Beltrami operator on smooth functions to the framework of differential forms on Riemannian manifolds, is a central analytic tool in spectral geometry revealing geometric and topological features of the underlying space through its interaction with homology. In this talk, we present geometric lower bounds for the smallest positive eigenvalue of the Hodge Laplacian in the class of non-convex domains given by Euclidean annular regions with a convex outer boundary and a spherical inner boundary. We also discuss extensions to perforated domains in general. The proofs employ local-to-global arguments via an explicit isomorphism between Čech cohomology and de Rham cohomology to obtain Poincaré-type inequalities with explicit geometric dependence, together with certain generalised versions of the Cheeger-McGowan glueing lemma.

Based on joint work with Pierre Guerini.

Keywords: Hodge Laplacian, Eigenvalue bounds, Differential forms.

Geometric Hardy inequalities on the Heisenberg groups via convexity

---

Marianna Chatzakou

Ghent University, Department of Mathematics: Analysis, Logic and Discrete Mathematics

We prove L^p-Hardy inequalities with distance-to-the-boundary potentials on the Heisenberg groups and, more generally, on step-two stratified groups. The guiding principle is a geometric condition expressed through horizontal p-superharmonicity of the distance function, which allows the sharp constant $((p-1)/p)^p$ to be recovered in several subelliptic settings. We discuss how this condition applies to Euclidean distance for convex and selected non-convex domains, to the pseudodistance associated with the fundamental solution of the sub-Laplacian on half-spaces and convex polytopes, and to the Carnot–Carathéodory distance on half-spaces and bounded convex domains. The results provide a unified convexity-based approach to geometric Hardy inequalities and extend known sharp inequalities beyond the classical Euclidean framework.

Based on joint work with G. Barbatis and A. Tertikas.

Keywords: Hardy inequalities, Heisenberg groups, Horizontal convexity.

Geometric (in)stability for the Maxwell's spectrum

Francesco Ferraresso

University of Verona, Italy

I will discuss a few results regarding the spectrum of the time-harmonic Maxwell system in domains with interesting geometry. For product domains, I will show that the classical TE-TM modes decomposition of the eigenvalues generalises to the "curved case", where the domain is the product of a two-dimensional manifold and an interval. In particular, for thin curved domains the eigenvalues of the Maxwell system converge to the eigenvalues of the Dirichlet Laplacian on the two-dimensional manifold, in the vanishing thickness limit. I will further present a few surprising spectral instability results in the unbounded setting, showing that the Maxwell spectrum is very sensitive to small perturbations.

Based on ongoing work with M. Marletta(Cardiff) and on a joint article with L. Provenzano(Sapienza Rome).

Keywords: spectral instability, Maxwell's equations.

Absolute continuity beyond one dimension: Sobolev, metric, and distortion perspectives

---

Anatoly Golberg

School of Mathematical Sciences, Holon Institute of Technology

Absolute continuity is a central notion in one-variable analysis, tightly connected with bounded variation, differentiability almost everywhere, and Lusin's condition. In higher dimensions, however, its correct analogue is far from unique. This talk surveys several multidimensional notions of absolute continuity AC^n and bounded variation BV^n, including the classes introduced by Malý, Hencl, Csörnyei, and Bongiorno, and explains their relations to Sobolev mappings, Lusin's (N)-property, area formulas, and absolute continuity on lines, paths, and surfaces. Particular emphasis is placed on mappings of finite distortion, finitely bi-Lipschitz mappings, and mappings of finite metric or area distortion. We will highlight key inclusions, counterexamples, and sharp phenomena showing why higher-dimensional absolute continuity is more subtle than its classical one-dimensional counterpart. The talk concludes with an open problem concerning the relation between AC^n, BV^n, and ACL.

Based on joint work with E. Afanas'eva.

Keywords: absolute continuity, bounded variation, finite Lipschitzness.

Well-posedness of a generalized Stokes operator on some non-compact manifolds via layer-potentials

---

Mirela Kohr

Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Romania

We study the generalized Stokes operator

$$\boldsymbol{\Xi} = \boldsymbol{\Xi}_{V,V_0} = \begin{pmatrix} \boldsymbol{L} + V & \nabla \\ \nabla^* & -V_0 \end{pmatrix}$$

on a domain with straight cylindrical ends Ω using the method of layer potentials on a larger manifold with straight cylindrical ends M, $\Omega \subset M$. The operator $\boldsymbol{\Xi} = \boldsymbol{\Xi}_{V,V_0}$ recovers the classical Stokes operator $\boldsymbol{\Xi}_{0,0}$ when the potentials V and V_0 vanish. Under suitable positivity assumptions at infinity on V and V_0, we prove that $\boldsymbol{\Xi}$ is Fredholm on $L^2(M; TM \oplus \mathbb{C})$. This allows us to define the single- and double-layer potential operators $\boldsymbol{S}$ and $\frac{1}{2}\mathbf{I} + \boldsymbol{K}$. Under further positivity assumptions at infinity, we prove that these operators are also Fredholm. Under slightly stronger assumptions on V and V_0 (including non-negativity everywhere), we prove the invertibility of the operators $\boldsymbol{\Xi}$, $\boldsymbol{S}$, and $\frac{1}{2}\mathbf{I}+\boldsymbol{K}$. The invertibility of these operators leads to well-posedness results for the (linear) Stokes boundary value problem with Dirichlet boundary conditions on domains with straight cylindrical ends. The proofs of these results require to develop an "algebra tool kit" to deal with limit and jump relations of the layer potential operators on manifolds with cylindrical ends. As an application of all these results, we prove the well-posedness of the Dirichlet problem for the generalized Navier-Stokes system with small data on a domain with cylindrical ends.

Joint work with Victor Nistor (Metz) and Wolfgang L. Wendland (Stuttgart).

Keywords: Stokes operator, non-compact manifolds, layer potentials.

Shape sensitivity of Neumann-Poincaré eigenvalues

Paolo Luzzini

Dipartimento di Scienze e Innovazione Tecnologica, Università del Piemonte Orientale

In this talk, we consider the eigenvalue problem for the integral operator known as the Neumann-Poincaré operator on the boundary of a perturbed $C^{1,\alpha}$ domain. We show that simple eigenvalues and the symmetric functions of multiple eigenvalues depend analytically upon domain perturbations, and we derive an explicit formula for their shape derivative that extends previously known results. We also present several consequences of our result and a number of open problems.

This talk is based on joint work with M. Dalla Riva, P.D. Lamberti and P. Musolino.

Keywords: Neumann-Poincaré operator, shape sensitivity analysis, eigenvalue problem.

Topological variations in the Einstein-Hilbert functional

Miltiadis Paschalis

Department of Mathematics, National and Kapodistrian University of Athens

We revisit the idea of topological variations, originally introduced by Wheeler and Hawking, from a rigorous perspective. Starting from a localized version of the Einstein-Hilbert variational principle, we encode the key aspects of the variational procedure in the form of a topology on a suitable space of Sobolev variational configurations, which is the final topology generated by the admissible variational maps. This framework naturally lends itself to generalization, and we rigorously introduce two distinct types of topological variations, corresponding to the infinitesimal addition of disconnected components and to infinitesimal surgeries, both motivated by related physical concepts. Using tools from the theory of Sobolev spaces and precise asymptotics, we establish dimensional obstructions for the continuity and differentiability of the Einstein-Hilbert action with respect to these variations, and show that in the extended variational framework the action does not admit critical points in dimension $n = 4$, while higher dimensions are free of this problem. Finally, we demonstrate the non-trivial effect of higher order curvature terms on the critical dimension.

Keywords: topological functional derivative, Einstein-Hilbert, infinitesimal surgery.

Estimates of the modulus of continuity of the logarithmic double layer potential

Sergiy Plaksa

Institute of Mathematics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine

For the real part of the Cauchy-type integral, which is known to be the logarithmic double layer potential, we obtain estimates of the modulus of continuity in the closure of domain bounded by an Ahlfors-regular integration curve. These estimates are more exact than the well-known Zygmund estimate for the modulus of continuity of the Cauchy-type integral. The accuracy of estimates for the logarithmic double layer potential is proved by constructing an example of a curve and an integral density for which the specified estimates are exact with respect to the order of smallness.

Based on joint work with Alexander Sarana.

Keywords: logarithmic double layer potential, Cauchy-type integral, Ahlfors-regular curve.

An isoperimetric inequality for the second Neumann eigenvalue on the sphere

Luigi Provenzano

Dipartimento di Scienze di Base e Applicate per l'Ingegneria, Sapienza Università di Roma, Italy

We prove that the geodesic disks are the unique maximisers of the fist non-trivial Neumann eigenvalue among all simply connected domains of the sphere $\mathbb{S}^2$ of fixed area.

Based on joint work with Alessandro Savo.

Keywords: Isoperimetric inequality, spherical domains, method of prescribed level lines.

Geometric properties of positive solutions of $-\Delta u = u^p$ on convex domains of $\mathbb{S}^2$

Daniel Raom

Dipartimento di Scienze di Base applicate all'Ingegneria, Sapienza University of Rome, Italy

We study positive solutions of the Dirichlet problem

$$\begin{cases} -\Delta u = u^p & \text{in } \Omega \subset \mathbb{S}^2, \\ \quad u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P_p}$$

where Ω is a uniformly geodesically convex domain with smooth boundary. In this talk, we show that for $0 \leq p < 1$, the unique positive solution of (P_p) is such that $u^{(1-p)/2}$ is strictly concave in Ω; for $1 < p \leq 3$, every positive solution of (P_p) is such that $u^{(1-p)/2}$ is strictly convex in Ω; for $p = 1$, we replace the right-hand side of (P_p) by $\lambda_1 u$, where λ_1 and u are respectively the first eigenvalue and the positive first Dirichlet eigenfunction of $-\Delta$ on Ω, and in this case it is well-known that u is strictly log-concave. Consequently, for $0 \leq p \leq 3$, any positive solution of (P_p) has strictly convex superlevel sets and a unique nondegenerate maximum. Time permitting, we discuss the generalization to geodesically convex domains of $\mathbb{S}^2$ and to dimensions $n \geq 3$.

Based on joint work with Luigi Provenzano and Massimo Grossi.

Keywords: 2-sphere, geodesically convex domain, power concavity.

Semiclassical Spectral Series of Schrödinger Operator with Delta-Potential on Surfaces of Revolution

---

Vladislav Rykhlov

Department of Mathematics and Mechanics, Lomonosov Moscow State University, Russia

We consider the spectral problem for the Schrödinger operator on surfaces of revolution, $M^2 \subset \mathbb{R}^3$ or $M^3 \subset \mathbb{R}^4$, of the form

$$H\psi = E\psi, \qquad H = -\frac{h^2}{2}\Delta + \delta_{x_1}(x) + \delta_{x_2}(x), \qquad x \in M, \tag{1}$$

where $-\Delta$ denotes the Laplace–Beltrami operator on M, and δ_{x_j} represents the Dirac delta function centered at x_j, in the semiclassical limit as $h \to +0$. The operator H is understood as a self-adjoint extension of the Laplace–Beltrami operator restricted to the functions from $W_2^2(M)$ vanishing at x_1 and x_2 (extensions of that kind can be parametrized by $U(2)$).

We obtain an explicit form of the quantization conditions that allow one to establish the asymptotic behavior of the spectrum of H. We derive a representation of eigenfunctions in terms of the Bessel and Neumann functions of zero order (two-dimensional case) or the Bessel functions of half-integer order (three-dimensional case). This representation offers a clear understanding of the structural characteristics and behavior of the eigenfunctions as $h \to 0$.

Based on joint work with Andrei Shafarevich.

Keywords: Semiclassical approximation, spectral problems, self-adjoint extensions.

On solution of a system of two R-linear conjugation problem

Sergei Rogosin

Department of Economics, Belarusian State University, Minsk, Belarus

The constructive solution of the double $\mathbb{R}$-linear conjugation problem with rational coefficients on the unit circle is obtained. The problem is equivalently reduced to four-dimensional $\mathbb{C}$-linear conjugation problem (the Riemann boundary value problem). To solve the latter problem, the factorization of the 4×4 matrix-coefficient is constructed using a generalization of the G.N Chebotarev's method. This method is based on the inductive approach reducing the factorization of a high order matrix to the factorization its lower order minor.

Based on joint work with Maryna Dubatovskaya.

Keywords: Double $\mathbb{R}$-linear conjugation problem, factorization, inductive approach.

Extremal problems for symmetric functions of the first three Maxwell eigenvalues

Rebecca Sempio

Department of Mathematics, University of Padova, Italy

It is known that the Faber-Krahn inequality does not hold for Maxwell eigenvalues: regardless of whether a volume or a perimeter constraint is imposed, the ball does not minimize the first eigenvalue. Notably, the first eigenvalue in the ball has multiplicity three. Motivated by this observation, as well as by a criticality result established by P. D. Lamberti and M. Zaccaron in 2021, we investigate symmetric functions of the first three Maxwell eigenvalues and discuss their potential minimization properties under suitable geometric constraints.

Based on joint work with Pier Domenico Lamberti and Luigi Provenzano.

Keywords: Maxwell eigenvalues, Shape optimization, Spectral inequalities.

Regularity of non-linear elliptic equations in metric spaces

Ivan Yuri Violo

Department of Mathematics, University of Pisa, Italy

Elliptic equations in metric spaces have attracted increasing attention over the last two decades, especially concerning the regularity of weak solutions. It is now well understood

that the De Giorgi-Nash-Moser theory can be extended to this setting, assuming only that the ambient space is doubling and supports a local Poincaré inequality. However, Lipschitz regularity might fail in this general setting.In this talk, I will show that, assuming a lower Ricci curvature bound in a weak sense, it is possible to obtain local Lipschitz regularity of solutions to a general family of quasilinear elliptic equations, including p-harmonic functions. The result is obtained using a Galerkin-type approximation method.

Based on joint work with Simon Schulz.

Keywords: elliptic regularity, metric spaces, p-harmonic functions.

A pointwise ergodic theorem along the primes

James Wright

School of Mathematics, The University of Edinburgh, UK

In joint work with M. Mirek and R. Wan, we extend recent work of Krause, Mousavi, Tao and Teravainen and establish a pointwise ergodic theorem along the primes for general multilinear, polynomial ergodic averages whenever the polynomials have distinct degrees.

Some of the key ingredients are: (1) a new, robust multilinear circle method for von Mangoldt weighted averages, (2) a new inverse theorem and multilinear Weyl estimates for Cramer-weighted averages and (3) a far-reaching arithmetic multilinear estimate over the profinite integers.

Local well-posedness of solutions of the Cauchy problem for the Korteweg-de Vries equation in weighted Sobolev

---

Lyailya Zhapsarbayeva

Department of Fundamental Mathematics, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan

We study the IVP for the Korteweg-de Vries equation

$$\begin{cases} \partial_t u + \partial_x^3 u + \partial_x(u^2) = 0, \ x \in \mathbb{R}, \ t > 0, \\ u(x,0) = u_0(x), \qquad x \in \mathbb{R}, \end{cases} \tag{2}$$

in the weighted Sobolev space $H^{3/4}(\mathbb{R}) \cap L^2(|x|^{2r}dx)$. Here $u = u(x,t)$ is a real-valued function.

We apply the Banach fixed point theorem to the integral equation version of IVP (2)

$$u = e^{it\partial_x^3}u_0 - \int_0^t e^{i(t-\tau)\partial_x^3}(\partial_x(u^2))(\cdot,\tau)\,d\tau, \tag{3}$$

where $e^{it\partial_x^3}$ is the semigroup associated with the linear part of (2).

Theorem. *Let $u_0 \in H^{3/4} \cap L^2(|x|^{2r}dx)$ for $r \in (0, 3/8)$. Then, there exist $T > 0$ and a unique solution u to the system of the integral equation* (3) *such that*

$$u(\cdot, t) \in H^{3/4} \cap L^2(|x|^{2r}dx), \quad t \in [0, T].$$

Based on joint work with Alejandro J. Castro Castilla.

Keywords: Weighted Sobolev space, Korteweg-de Vries equation, local well-posedness.