Weak solutions for the wave equation on a time-dependent domain

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In this talk we present a notion of weak solutions for the wave equation on a time-dependent domain with homogeneous Dirichlet boundary value data and standard initial conditions. Under regularity assumptions of the time evolution of the space domain, existence can be proven via penalization argument (cf. [4]) or via time discretization and approximation argument by cylindrical domains.

Moreover, in the case of a $C^{1,1}$-evolution, by means of suitable diffeomorphisms, the problem can be recast into an hyperbolic equation on a fixed domain: we prove the equivalence of the notions of weak solutions of the two problems.

Finally, this equivalence allows us to obtain an energy balance, following a double regularization argument in the spirit of [1] and [3], and to discuss better regularity properties depending on the domain and the initial data.

This talk is based on a joint work with G. Lazzaroni, F. Riva, and F. Solombrino (cf. [2]).

Keywords: hyperbolic equation in time-dependent domains, energy balance, debonding model.

References

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