The pseudo-orthogonality for graph 1-Laplacian eigenvalues and applications to higher Cheeger constants and data clustering

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Abstract. The data clustering problem consists in dividing a data set into prescribed groups of homogeneous data. This is a NP-hard problem that can be relaxed in the spectral graph theory, where the optimal cuts of a graph are related to the eigenvalues of graph 1-Laplacian. In this paper, we give new notations to punctually describe the paths, among critical eigenvectors of the graph 1-Laplacian, realizing sets with prescribed genus. We introduce the pseudo-orthogonality to characterize the special eigenvalue $m_3(G)$ for the 1-Laplacian on graphs. Furthermore, we show that $m_3(G) \geq h_3(G)$, the third graph Cheeger constant. This is a first step for proving that the k-th Cheeger constant is the minimum of the 1-Laplacian Raylegh quotient among vectors that are pseudo-orthogonal to the vectors realizing the previous $k-1$ Cheeger constants. Eventually, we apply these results to give a method and a numerical algorithm to compute $m_3(G)$, based on a generalized inverse power method.

Based on joint work with Antonio Corbo Esposito.