Schauder type estimates for degenerate Kolmogorov equations with Dini continuous coefficients

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We study the local regularity of the classical solution u to $\mathcal{L}u = f$, where $\mathcal{L}$ is the second order linear differential operator $\mathbb{R}^{N+1}$:

$$\mathcal{L}u := \sum_{j,k=1}^{N} a_{jk}\partial^2_{x_j x_k} u + \sum_{j,k=1}^{N} b_{jk} x_k \partial_{x_j} u - \partial_t u\,.$$

Here, $A = (a_{jk})_{j,k=1,\ldots,N}$, $B = (b_{jk})_{j,k=1,\ldots,N}$ are real valued matrices with constant coefficients, with A symmetric and strictly positive. In particular, we prove that, if the operator $\mathcal{L}$ satisfies Hörmander's hypoellipticity condition and f is a Dini continuous function, then the second order derivatives of the solution u to the equation $\mathcal{L}u = f$ are Dini continuous functions as well. We also consider the case of Dini continuous coefficients a_{jk}'s. The proof of our main results is based on the blow-up technique introduced by Wang in [4] in the study of the Poisson equation. In order to adapt Wang's method to our case, we generalize the results by Bonfiglioli [1] and those by Pagliarani, Pascucci e Pignotti [2] on the Taylor polynomial. In fact, while the authors of the above articles suppose that the second order derivatives of the function u are Hölder continuous, here we do not assume extra conditions on those derivatives and we prove the existence of the Taylor polynomial for u under these assumptions. This is a joint project [3] in collaboration with S. Polidoro and B. Stroffolini.

References

[1] Andrea Bonfiglioli. Taylor formula for homogenous groups and applications. *Mathematische Zeitschrift*, 2008.

[2] Stefano Pagliarani, Andrea Pascucci, and Michele Pignotti. Intrinsic Taylor formula for Kolmogorov-type homogeneous groups.*Journal of Mathematical Analysis and Applications*, 435, 01 2015.

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[4] Xu-Jia Wang. Schauder estimates for elliptic and parabolic equations*. *Chinese Annals of Mathematics, Series B*, 27:637–642, 01 2006.