Solution to $\mathbb{R}$-linear conjugation problem with rational coefficients

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It is proposed a method of solution to $\mathbb{R}$-linear conjugation problem (known also as Markushevich boundary value problem) with rational coefficients on the unit circle:

$$\varphi^+(t) = a(t)\varphi^-(t) + b(t)\overline{\varphi^-(t)} + f(t), \quad t \in \mathcal{L}. \tag{1}$$

It is based on an approach, which is recently proposed by the authors [5], which in turn is related to Chebotarev algorithm of factorization of matrix-functions [2]. This problem is important due to its connection to a number of deep mathematical questions as well as to numerous application (in particular, in the theory of composite materials, see [4]).

Factorizing (see [3]) the main coefficient

$$a(t) = \chi^+(t)t^{æ}\chi^-(t), t \in \mathbb{T},$$

after a series of simple transformations we arrive at the equivalent form of the boundary condition

$$\psi^+(t) = t^{æ}\psi^-(t) + p(t)\overline{\psi^-(t)} + h(t), \quad t \in \mathbb{T}, \tag{2}$$

where $p(t)$ is a boundary value of the rational function analytic outside of the unit disc.

In the above notation problem (2) can be equivalently reduced to the vector-matrix $\mathbb{C}$-linear boundary value problem

$$\Psi^+(t) = \begin{pmatrix} t^{æ} & 0 \\ 0 & t^{æ} \end{pmatrix} \begin{pmatrix} 1 - p(t)\overline{p(t)} & p(t) \\ -\overline{p(t)} & 1 \end{pmatrix} \Psi^-(t) + r(t), \ t \in \mathbb{T}. \tag{3}$$

Solution to this problem is equivalent to the solution of factorization problem of the matrix coefficient (see [6]).

We propose an efficient method of factorization of rational matrix functions which consists of two-fold application of a generalization of Chebotarev's algorithm. First, the matrix coefficient is transformed to the triangular form by using a series of multiplication on the polynomial matrices of the unit determinant. Second, the triangular matrix is factorized by the similar transformation. The algorithm is illustrated by a series of examples. The proposed algorithm is much easy to apply with respect to known algorithm of factorization of rational matrix functions [1, 6].

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References

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