Spectra of the Steklov- and Robin-Laplace-problems in bounded, cuspidal domains

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It is well-known by works of several authors that the spectrum of the Neumann-Laplace operator may be non-discrete even in bounded domains, if the boundary of the domain has some irregularities. In the same direction, in a paper in 2008 with S.A. Nazarov we considered the Steklov spectral problem in a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with a peak and showed that the spectrum may be discrete or continuous depending on the sharpness of the peak. Later, we proved that the spectrum of the Robin Laplacian in non-Lipschitz domains may be quite pathological since, in addition to countably many eigenvalues, the residual spectrum may cover the whole complex plain.

We have recently complemented this study in two papers, where we consider the spectral Steklov- and Robin-Laplace-problems in a bounded domain Ω with a peak and also in a family Ω_ε of domains blunted at the small distance $\varepsilon > 0$ from the peak tip. The blunted domains are Lipschitz and the spectra of the corresponding problems on Ω_ε are discrete. We study the behaviour of the discrete spectra as $\varepsilon \to 0$ and their relations with the spectrum of case with Ω. In particular we find various subfamilies of eigenvalues which behave in different ways (e.g. "blinking" and "stable" families") and we describe a mechanism how the discrete spectra turn into the continuous one in this process.

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