Tikhomirov V. (Moscow University, Russia)

Kolmogorov-type inequalities for intermediate derivatives and the Lagrange Principle in the theory of extremum

In the course of three lectures, a collection of Kolmogorov-type inequalities for intermediate derivatives (the list of them see below) will be analyzed by a unique general principle of the theory of extremal problems. This principle is called the Lagrange principle for necessary conditions.

According to this principle, when one searches a necessary condition of an extremal problem with equality constraints in which smoothness is interlaced with convexity, it is sufficient to construct the Lagrange function of the problem and then to apply necessary conditions for a minimum of the Lagrange function as if the variables are independent. The equations that one will find, combined with the given equations, will serve to determine all the unknowns.

In the first lecture the Lagrange principle for one general class of extremal problems will be proved. Necessary conditions of extremum of Euler, Lagrange, Poisson and others, in the calculus of variations, theorems of Kuhn–Tucker and similar to them conditions of minima for convex problems, Pontryagin maximum principle for problems of optimal control and many other well known necessary conditions are corollary of this result.

Consider the following collection of Kolmogorov-type inequalities:

$$\|x^{(k)}(\cdot)\|_{L_q(T)} \to \max,\ \|x(\cdot)\|_{L_p(T)} \le 1,\ \|x^{(n)}(\cdot)\|_{L_r(T)} \le 1. \qquad (P(k,n,p,q,r,T)$$

In the second and the third lectures, the Lagrange principle will be applied to the following problems of this family

1. $(P(1,2,\infty,\infty,\infty,\mathbb{R}_+)$ (Landau, 1913),
2. $(P(1,2,\infty,\infty,\infty,\mathbb{R})$ (Hadamard, 1914),
3. $(P(k,n,\infty,2,2,2,\mathbb{R})$, 4. $(P(1,2,2,2,2,\mathbb{R}_+)$ (Hardy, Littlwood, Polya, 1934),
5. $(P(k,n,\infty,\infty,\infty,\mathbb{R})$ (Kolmogorov, 1938),
6. $(P(0,1,p,q,r,\mathbb{R}_+)$ (Nagy, 1941)
7. $(P(k,n,1,1,1,\mathbb{R})$ (Stein, 1957)
8. $(P(k,n,2,2,2,\mathbb{R}_+)$(Lyubich, 1960, Kupzov, 1975)
9. $(P(1,2,\infty,\infty,r,\mathbb{R}_+)$(Arestov, 1972)
10. $(P(k,n,2,2,2,\mathbb{R})$ (Gabushin, 1969, Kalyabin, 2007)
11. $(P(1,2,2,\infty,\infty,\mathbb{R}_+)$ (Fuller, 1960, Gabushin, 1969, Magaril-Il'yaev, 1883)
12. $(P(p,\infty,1,\mathbb{R}_+)$(Magaril-Il'yaev, 1883)

Some new problems also will be considered.

Bibliography

1. G. G. Magaril-Il'yaev and V. M. Tikhomirov, Kolmogorov-type inequalities for derivatives, Mat. Sbornik 188, 12, 73–106; English transl. in Mathematics 188, 12, 1799–1822

2. G. G. Magaril-Il'yaev and V. M. Tikhomirov, Convex Analysis: theory and applications; English transl. in AMS, Transl. Math. Monogr, Vol 222, 2003