New Frontiers of Celestial Mechanics: theory and applications

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Abstracts

Alain Albouy (IMCCE, Paris)

The Levi–Civita regularizations

Levi–Civita obtained in 1904 a regularizing transformation of the planar restricted 3–body problem. Sundman's results in 1909 indicated that a similar transformation should exist without such restrictions, i.e., for the full 3–body problem in space. Levi–Civita published such a transformation in [1]. It is very different from the previous one, but both have some relation with the Darboux inversion (1889). Levi–Civita insisted that he did not find this transformation easily. I will try to explain the difficulties, helped by a remark by J. Moser about this transformation (1970).

Our bibliographic analysis appeared in [2]. We wish to thank G. Gronchi for mentioning us that Levi–Civita's papers from 1915 were misread.

[1] T. Levi–Civita, Sopra due trasformazioni canoniche desunte dal moto parabolico, Atti della Reale Accademia Dei Lincei, Rendiconti, serie 5, v. 25 (1916), 445–458

[2] A. Albouy, L. Zhao, Darboux inversions of the Kepler problem, Regular and Chaotic Dynamics, 27 (2022), 253–280

Adrian Bustamante (Università Tor Vergata)

Breakdown of Tori in low and high dimensional conservative and dissipative standard maps

We study the breakdown of rotational invariant tori by implementing three different methods. First, we analyze the domains of analyticity of a torus with given frequency through the computation of the Lindstedt series expansions of the embedding of the torus. Then, we implement a Newton's method to construct the embedding of the torus; the breakdown threshold is then computed by looking at the blow–up of the Sobolev's norms of the embedding. Finally, according to Greene's method, we estimate the breakdown threshold of an invariant torus with irrational frequency by looking at the stability of the periodic orbits with periods approximating the frequency of the torus. We apply these methods to 2–dimensional and 4–dimensional standard maps. The 2–dimensional maps can either be conservative (symplectic) or dissipative (precisely, conformally symplectic). The 4–dimensional maps are obtained coupling (i) two symplectic standard maps, or (ii) two conformally symplectic standard maps, or (iii) a symplectic and a conformally symplectic standard map. While Padé and Newton's methods perform quite well and provide reliable results, when applying Greene's method, the computation of the periodic orbits in higher dimensional, dissipative maps is particularly complex. This is a joint work with A. Celletti and C. Lhotka.

Nelson Callegari (São Paulo State University)

A Hamiltonian for 1:1 Rotational Secondary Resonances of Inner Small Satellites

Consider a rigid body with semi–axes $a > b > c$ along the x, y, z axes and the corresponding moments of inertia $A < B < C$. A key parameter is $\varepsilon = [3(B - A)/C]^{1/2}$, which jointly to the orbital eccentricity, dictate the phase space morfology of the rotational phase space. (We suppose the satellite rotating around the z axis only and suffering time dependent torque on its rotation due to the primary body.) In the case of satellites with quasi-spherical shape and small eccentricity, the main structure observed is the synchronous resonance (SR), characterized by a unique stable center where the rotation rate of the secondary equals its mean-motion. It is known since Voyager images that the giant planets and Mars have groups of small close-in satellites with tens of km in diameter at most. In general, these satellites are irregularly shaped and, due to this, ω_0 is close to the unit for several of them. The rotational dynamics within the SRs in the case of such sort of satellites show much more complex structures, in spite of very small eccentricity of their orbits in general. For instance, Prometheus and Pandora ($e < 0.004$) show period-doubling bifurcation modes (PDBM) (e.g. Melnikov and Shevchenko 2008). Amalthea and Prometheus show additional structure located in the interior of SR which is designated by β-regime, and there are two stable centers associated with synchronism (the alfa and beta regimes; (see Melnikov and Shevchenko 2008 and references therein). Hyperion is the famous example where large chaotic layer occupies the SR (Wisdom et al. 1984), but in this case $e \sim 0.1$.

The works cited above apply in general numerical techniques like surfaces of section aiming to map stability and chaos. In the case of the α and β-regime, no quantitative explanation is given in literature. Wisdom (2004) formulated a Hamiltonian modeling for a 3:1 secondary resonance (SER) near the SR of Enceladus. SER involves commensurabilities between the frequency of the physical libration and the mean-motion. Following the Wisdom's analytical methodology, in this work we show that the α and β-regime in the cases of Prometheus and Amalthea can by modeled by a $1:1$ secondary resonance, with the SR.

Thomas and Helfenstein (2020) give the values of a, b, c for many Saturnian satellites observed by Cassini spacecraft. Utilizing the published parameters of Methone, in this work, we detect the alfa and beta regimes for the satellite and modeled them with our Hamiltonian approach for SER.

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[2] Wisdom, J., Peale, S. J., Mignard, F.. The chaotic rotation of Hyperion. Icarus, 58, 137-152 (1984).

[3] Wisdom, J.. Spin-Orbit Secondary Resonance Dynamics of Enceladus. AJ, 128, 484-491 (2004).

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Chiara Caracciolo (Uppsala University)

Long–term stability of planetary systems: some computer–assisted result

The planetary three-body problem with realistic values of the masses and of the orbital parameters is a famous example of a nearly-integrable Hamiltonian system with a non-negligible perturbative parameter. KAM theory provides algorithms to study the stability of this system. The applicability of the analytical results can be significantly improved when these algorithms are explicitly implemented. In this talk, I will discuss how to build a rigorous computer-assisted proof out of these semi–analytical methods. I will present some application of these techniques to prove the long–term stability in a secular planetary three-body problem. The presentation is based on joint works with U. Locatelli, M. Sansottera and M. Volpi.

Alessandra Celletti (Università Tor Vergata)

The Earth's satellite dynamics

The study of the dynamics of a satellite moving around the Earth (either an artificial satellite or a space debris) is studied through different analytical and computational tools, which range from classical perturbation theory to machine learning techniques. We consider a model including the gravitational attraction of Earth, Sun, Moon and the Solar radiation pressure. Through a sequence of hierarchical Lie series normalizations (since the model depends on variables with different rates of variation), we compute the so–called proper elements, which are quasi–integrals of motion. We simulate break–up events of satellites and analyze the mean and proper elements of the generated fragments using statistical data analysis and machine learning methods. We also test our results on real sample cases.

Ariane Courtot (IMCCE, Paris)

Chaos in meteor showers: the example of Geminids, Draconids and Leonids

Meteoroids have peculiar dynamics owing to their relatively high non-gravitational forces and their multiple close encounters. When a meteoroid stream meets with the Earth, a meteor shower is produced.

Today more than 900 meteor showers are listed by the IAU, meaning a similarly large number of comet-like parent bodies existed in the Earth vicinity in the near past (1–100kyrs). This raises the question of the authenticity of these showers. To tackle this, we aim to better understand the dynamical evolution of meteoroids, which can be done by drawing chaos maps.

In previous works, we used chaos maps to study the Geminid meteor showers (Courtot et al., in revision). Those maps revealed the importance of mean-motion resonances (MMR) with the Earth in the chaoticity of the Geminids. We also investigated the effect of non-gravitational forces and how it competes with the effect of MMR for small particles. In this talk, we will present our new results: we drew chaos maps on the Draconids and the Leonids meteor shower. The effect of MMR with Jupiter will be shown, as well as the effect of close encounters, mainly with Jupiter, Saturne and the Earth. A first exploration of the impact of non- gravitational forces will also be presented.

Veronica Danesi (Università Tor Vergata)

Secular invariant tori for extrasolar systems in MMR: application to HD60532.

We investigate the long-term dynamics of HD60532, an extrasolar system hosting two giant planets orbiting in a 3:1 mean motion resonance. We consider the secular approximation at order one in the masses which results (after the reduction of the constants of motion) in a resonant Hamiltonian with two libration angles. In this framework, the usual algorithms constructing the Kolmogorov normal form approach do not easily apply and we need to perform some untrivial preliminary operations, in order to adapt the method to this kind of problems. First, we perform an average over the fast angle of libration which provides an integrable approximation of the Hamiltonian. Then, we introduce action-angle variables that are adapted to such an integrable approximation. This sequence of preliminary operations brings the Hamiltonian in a suitable form to successfully start the Kolmogorov normalization scheme. The convergence of the KAM algorithm is proved by applying a technique based on a computer-assisted proof. This allows us to reconstruct the quasi-periodic motion of the system, with initial conditions that are compatible with the observations. This work is made in joint collaboration with U. Locatelli and M. Sansottera.

Francesco Fassò (Università di Padova)

Nekhoroshev theory for perturbations of central forces

The talks reviews some known and some new results on the dynamics of a point in a small perturbation of a central force field in 3-space in the realm of Nekhoroshev theory for superintegrable systems, and with a focus on the appearance of chaoticity in resonance.

Luis Garcia–Naranjo (Università di Padova)

Symplectic reduction in celestial mechanics. A new perspective.

The reduction of differential equations concerns the decrease of the number of phase parameters in the presence of continuous symmetries. In the Hamiltonian context, such process can be conveniently combined with the restriction of the system to the level set of the momentum first integrals whose existence is guaranteed by Noether's theorem. In modern mathematical language, such combination is embodied in the celebrated process of "symplectic reduction" (Meyer 1973, Marsden–Weinstein 1974). In this talk I will present a novel interpretation of symplectic reduction which, among other things, sheds light in some classical and recent developments in celestial mechanics. This work is in collaboration with F. Fassò.

Clara Grassi (Università di Pisa)

Revisiting the computation of the critical points of the squared distance between two ellipses with a common focus

Computing the critical points of the squared distance between two ellipses with a common focus is of great interest in Celestial Mechanics, in particular in relation to the computation of the MOID (minimum orbit intersection distance), that is the distance between the two ellipses. This is useful in various applications, from impact monitoring of near-Earth asteroids to the detection of conjunctions between space debris. Different methods to compute all the critical points have been developed, for example in [4, 3, 1]. Starting from the expression of the squared distance in terms of the true or the eccentric anomalies of the two objects, we develop new methods, or revisit known ones, using the resultant theory. In each case, we obtain systems of two either ordinary or trigonometric bivariate polynomials and then apply the resultant theory to look for solutions. The possibility of using Chebychev's polynomials to improve the numerical stability is also explored. With one of the methods we are able to obtain the same trigonometric polynomial introduced in [3], using the resultant theory instead of Groebner's basis theory. For the different methods, we also test the reliability of the computed solutions using the results of [2] on the maximum MOID in the case of one circular and one elliptic orbit. Finally, we provide a comparison of the accuracy of each method and discuss the computational efficiency.

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Giovanni Federico Gronchi (Università di Pisa)

An overview of IOD methods

We review some recent results in the computation of preliminary orbits (IOD methods), that are used as initial guesses for the differential corrections algorithm.

The recent improvements in the observational technology and the introduction of new observables (e.g. radar data) stimulated the research in this direction, giving rise to new mathematical problems. We will focus in particular on the algebraic aspects, and we will also speak about some open problems.

Kathleen Howell

TBA

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Giacomo Lari (Università di Pisa)

Results of the Juno radio science experiment obtained with the Orbit14 software

NASA's Juno space mission is orbiting around Jupiter since 2016, collecting different kinds of data which have been crucial to improve our knowledge of the planet. In particular, the radio science experiment provides extremely accurate observations of the position and velocity of the Juno spacecraft, and precise orbit determination allows to estimate several physical and dynamical parameters of Jupiter. In this talk, we first introduce the orbit determination software Orbit14 and we describe the precise dynamical and observation models we implemented in the code. Then, we show the results of the fit of the Juno's radio science data and we present the obtained estimation of the gravitational field, tidal response and spin-axis precession of Jupiter.

Christoph Lhotka (Università Tor Vergata)

Spatial motion of charged dust in the solar system

The interaction of charged dust with interplanetary magnetic fields triggers latitudinal oscillations on secular time scales. We provide a simple mathematical model that allows to predict the center point of libration, libration frequencies, and the extend of these kind of oscillations based on simplified magnetic field models. We provide conditions for linear stability of steady state solutions, and discuss the geometry of the phase space of this interesting dynamical model. Our results are applicable to resonant kinds of motions of charged particles, in mean anomalies with a perturbing planet. Inside and outside resonance, latitudinal variations of charged dust in orbital inclinations exist, but are affected by the presence of an additional planet. We provide results for 1:1 mean motion resonances with planet Jupiter, and planet Venus. The applicability of our results for the distribution of charged dust in the solar system Is discussed on the basis of recent observations.

Xiadong Lu (Politecnico di Milano)

Reachable domain analysis for analytical design of end-of-life disposal

The interest towards post mission disposal for spacecrafts is increasing among both academic and industrial sectors due to the growing number of space debris. The analysis of dynamical behaviour of spacecraft orbits is a vital part in the design of post mission disposal. In this research, the analytical expression of the double averaged disturbing potential due to and lunisolar perturbations is obtained, in both geocentric equatorial frame and geocentric ecliptic frame. Pros and cons of the two frames for orbital perturbation problem is discussed. Assuming a circular Earth's orbit and a circular Moon's orbit, the phase space representation of the dynamics is formulated. It follows that the dynamical behaviour in the phase space of Keplerian elements is analysed and a reachable domain analysis of a single impulsive manoeuvre is carried out in the phase space using Gauss equations of finite-difference form. Based on the reachable domain analysis, analytical schemes for computing end-of-life disposal manoeuvres could be developed targeting a specific phase space region. A case study of Highly Elliptical Orbits (HEO) could then be carried out, in which the end-of-life disposal is targeting an atmospheric re-entry and the target region in phase space is defined by re-entry condition of the eccentricity.

Marcelo Domingos Marchesin (UFMG, Belo Horizonte)

A Family of Linear Stable Equilibria in the Sun-Earth-Sail Problem

The collinear libration point of the Sun-Earth Circular Restricted Three-Body Problem (CR3BP), L_3 is located opposite to the Earth with respect to the Sun. Whereas several space missions have been launched to the other two collinear equi- librium points, i.e., L_1 and L_2, the region around L_3 is so far unexploited essentially because of the severe communication limitations caused by Sun's blocking location. By using an adequate size, location and attitude of a solar sail, the equilibrium point can be displaced from its original location to allow direct communication between the satellite and Earth. This paper presents several families of artificial equilibria located on the semi-space which is permanently opposite to Earth in relation to the Sun, but which allows direct communication with Earth. We present a family of such equilibria which are linearly stable and therefore very useful for space missions.

Catello Leonardo Matonti (Politecnico di Torino)

Design and Comparison of Relative Dynamics Models about Cislunar Near Rectilinear Halo Orbits

This work presents a comparison between relative models of the Earth-Moon non-Keplerian dynamics regimes, in the perspective of modeling the close relative dynamics between two orbiting objects and of designing Guidance, Navigation and Control algorithms for space debris avoidance operation of future space exploration missions on cislunar Near Rectilinear Halo Orbits. According to the Artemis missions program, NASA will bring back humankind to the Moon in the next years thanks to the Gateway space station. In the Earth-Moon three body environment near the L1-L2 lagrangian points, the classic two body relative models are no more suitable, since the Moon gravitational influence is about 25-40% of the total one acting on a spacecraft. This raises the necessity of developing different relative models for space debris monitoring and safety applications. The work is distinguished for formulating a common benchmark for comparison of Restricted Three Body Problem (R3BP) and Restricted Two Body Problem (R2BP) relative models, showing the range of applicability and their limitations analyzed in terms of displacement with respect to an ephemeris propagation. Quantitative results are then provided to verify models performance at different space debris avoidance location along the orbit. Furthermore, a new R2BP relative model is developed with an innovative strategy which use local osculating Keplerian trajectories respect to a fictitious planet, showing an optimal trade-off between dynamics simplicity and relative errors accuracy. Finally, since the evolution of the relative geometry between Sun, Earth and Moon has a non-negligible influence, an epoch sensibility analysis is conducted by comparing several propagations with different revolutions of the reference orbit in an 18 years Saros Period. R2BP and R3BP models error oscillation behavior due to a change in epoch is then discussed.

Federico Mogavero (IMCCE, Paris)

Timescales of chaos in the inner Solar System: Lyapunov spectrum and quasi-integrals of motion

Numerical integrations of the Solar System reveal a remarkable stability of the orbits of the inner planets over billions of years, in spite of their chaotic variations characterized by a Lyapunov time of only 5 million years and the lack of integrals of motion able to constrain their dynamics. To open a window on such long-term behavior, we compute the entire Lyapunov spectrum of a forced secular model of the inner planets. We uncover a hierarchy of characteristic exponents that spans two orders of magnitude, manifesting a slow-fast dynamics with a broad separation of timescales. A systematic analysis of the Fourier harmonics of the Hamiltonian, based on computer algebra, reveals three symmetries that characterize the strongest resonances responsible for the orbital chaos. These symmetries are only broken by weak resonances, leading to the existence of quasi-integrals of motion that are shown to relate to the smallest Lyapunov exponents. A principal component analysis of the orbital solutions independently confirms that the quasi-integrals are among the slowest degrees of freedom of the dynamics. Strong evidence emerges that they effectively constrain the chaotic diffusion of the orbits, playing a crucial role in the statistical stability over the Solar System lifetime.

Alessandro Morbidelli (Observatoire de la Côte d'Azur))

Formation and evolution of super–Earth systems

Super–Earths seem to be the most ubiquitous planets in the galaxy and are found close to about half of the stars. They often appear in systems of planets of comparable radii (the so–called peas–in–the–pot pattern: Weiss et al. 2018, ApJ 155) on quasi–circular and coplanar orbits. Several models of their formation exist, which will be briefly reviewed. All realistic models predict that super–Earths form during the lifetime of gas in the protoplanetary disk, which is confirmed by the observations that basically all super–Earths that are not strongly irradiated by the central star have atmospheres of H and He (Fulton et al. 2017, AJ 154). Formation in a disk of gas implies planet migration until the inner edge of the disk and the formation of compact resonant chains. Some of these chains are indeed observed (Goldberg and Batygin, 2021, AJ 162) but most super–Earth systems don't show resonant relationships. Izidoro et al. (2017, MNRAS 470) showed that the observed orbital distribution of super–Earths can be explained if about 90% of the original resonant chains become dynamically unstable after the disappearance of the gas from the disk. In a series of numerical experiments Matsumoto et al. (2012, Icarus 121) indeed showed that compact resonant chains of several planets tend to become unstable. An explanation for the spontaneous onset of these instabilities has been provided by Pichierri and Morbidelli (2020, MNRAS 494) and involves secondary resonances between the resonant libration periods and the synodic periods between adjacent planets. This explanation has been recently confirmed in Goldberg et al. (2022, Icarus, in press), which showed that it leads to an instability criterion that reproduces the Matsumoto et al. (2012) numerical results. A small change in the mass of some planets, such as those due to the photoevaporation of the atmosphere of the planets which are the closest to the central star, can easily drive a planet resonant chain through a dynamical instability (Matsumoto and Ogihara, 2020, ApJ 893). Other dynamical instability mechanisms will be reviewed.

Rafael Ortega (University of Granada)

The spin-orbit problem, instability for large eccentricities and 1:1 resonance

A satellite is moving aroung a planet in a Keplerian orbit and the shape of the satellite is almost spherical. We follow the unique motion of the spin axis of the satellite that is symmetric and periodic, with the same period as the Keplerian orbit. For small eccentricities there is stability but for large eccentricities the motion becomes unstable. These facts are shown by numerical simulations and it is also possible to produce rigorous proofs.

Ernesto Perez–Chavela (ITAM, Mexico)

A new method to study relative equilibria on the sphere S^2

The simplest solutions of the N–body problem are those where the mutual distances among the masses remain constant for all time, that is, the mo- tions behave as a rigid body.

For N = 3 on the Euclidean space it is well known that there are exactly five relative equilibria: three collinear (Euler relative equilibria) and two planar forming an equilateral triangle (Lagrange relative equilibria). The big difficulty to study relative equilibria on the sphere S2, that we call RE by short, is the absence of the center of mass as a first integral, since many of the standard methods used in the classical case don't apply any more. Without the center of mass we do not know how to determine the rotation axis. In this talk I will show a new geometrical method to study RE on the sphere, when the masses are moving under the influence of a general attrac- tive potential. First we prove the existence of two new integrals of motion, which can be seen as an extension of the center of mass. These two new integrals allow us determine the rotation axis. Our method works for the general N–body problem on the sphere, but for simplicity in the computations, we restrict our analysis to the case N = 3. Applying our method, we give some new families of Euler and Lagrange RE on the sphere for the cotangent potential (the natural extension of the Newtonian potential to the sphere). Joint work with Toshiaki Fujiwara

Gabriella Pinzari (Università di Padova)

Quantitative KAM theory, with an application to the three–body problem

The hamiltonian of the three–body problem reveals, in a small domain, an equilibrium with a hyperbolic or elliptic character, according to the system of coordinates which is used. We discuss that a well designed KAM theory allows to infer the existence of both maximal and whiskered quasi–periodic motions in such domain.

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Melaine Saillenfest (IMCCE, Paris)

Oblique rings as a natural end state of migrating exomoons

Moons alter the spin-axis precession rate of their host planet. The tidal migration of moons is therefore an efficient driver of resonance crossing between the spin-axis and orbital precession modes of a planet. We examine the relevance of this mechanism for the exoplanet HIP41378 f, and show that an ancient satellite with roughly the mass of our Moon would be able to tilt the planet to nearly 90°. At this point, the moon becomes unstable and may be disrupted into a ring of debris. In transit data, such an oblique ring can be misinterpreted as an extended atmosphere. This phenomenon is a viable alternative to the – otherwise unexplained – apparent very low density of HIP41378 f.

Erica Scantamburlo (Università di Pisa)

Initial orbit determination from one position vector and a very short arc of optical observations

We address the problem of computing an asteroid orbit from one topocentric position vector $\mathcal{P}_1 = (\rho_1, \alpha_1, \delta)$, where ρ_1, α_1 and δ_1 denote respectively the topocentric distance, the right ascension and the declination at epoch t_1, and a very short arc (VSA) of optical observations, giving an attributable $\mathcal{A}_2 = (\alpha_2, \delta_2, \dot{\alpha}_2, \dot{\delta}_2)$ at the mean epoch $\bar{t}_2$ of the VSA.

Using the conservation laws of the angular momentum, the Laplace-Lenz vector and the energy in the two-body dynamics, we can write a system of polynomial equations in the unknowns $\dot{\rho}_1, \dot{\alpha}_1, \dot{\delta}_1, \rho_2, \dot{\rho}_2, z_2$, where z_2 is an auxiliary variable.

We prove that this system is consistent, i.e. it generically admits solutions (at least in the complex field), and we can obtain a univariate polynomial $\mathfrak{u}$ of degree eight in the unknown range ρ_2 at epoch $\bar{t}_2$ allowing to compute the solution of the orbit determination problem (ODP). Through Groebner bases theory, we can prove that the degree of $\mathfrak{u}$ is minimum among the degrees of all the univariate polynomial equations in ρ_2 that are consequences of the considered multivariate polynomial system.

The proposed method can be applied to several ODPs, such as the determination of the orbit of an Earth satellite when mixed (i.e. radar and optical) observations are available, the detection of impulsive maneuvers performed by an Earth satellite, and the initial orbit determination for an asteroid having a close encounter with the Earth, if this can be modelled with an instantaneous change of velocity.

Vladislav Sidorenko (Russian Academy of Sciences)

Adiabatic approximation in studies of co-orbital motions

We consider a system consisting of a star and two planets in co-orbital motion. Since the co-orbital motion corresponds to 1:1 mean motion resonance (MMR), it can be studied by methods developed to investigate resonance effects. One possible approach is to apply the adiabatic approximation proposed by J.Wisdom.

Such an approach implies the use of an approximate integral of the equations of motion for the analysis of the secular evolution. In order to construct the approximate integral for the system at MMR, it is necessary to write down the equations characterizing the variation of the resonance phase. Then these equations can be interpreted as the equations describing the dynamics of 1DOF Hamiltonian system which depends on slower variables as parameters.

The value of its "action" variable will be an adiabatic invariant, i.e., it will be an approximate integral we need.

Proceeding in this way, we have classified the possible modes of co-orbital motion in the planar unrestricted three-body problem and established the conditions for transitions between these modes.

Menios Tsiganis (Aristotle University of Thessaloniki)

Interpreting DART in view of Hera: post-impact dynamics of the Didymos system

The impact of the DART spacecraft on asteroid Dimorphos - the smaller of the two components of the Didymos system - on September 26, 2023 marked the first - successful - attempt of humanity to alter the trajectory of a natural celestial object. As was announced a few days later, the impact reduced the orbital period of Dimorphos by about 33 minutes. As new information about the post-impact orbit and the objects' characteristics come along, interpreting the post-impact dynamics of the system and predicting what the ESA mission - Hera - will observe in 2026 becomes relevant. In this talk I will describe results coming from numerical and analytical models used in these studies. Additionally, I will describe the intense efforts of the international campaign for observing stellar occultations by Didymos and the significance of its results.

Antonio Ureña (University of Granada)

On the Lambert problem with drag

The Lambert problem consists in connecting two given points in a given lapse of time under the gravitational influence of a fixed center. While this problem is very classical, we are concerned here with situations where friction forces act alongside the Newtonian attraction. Under some boundedness assumptions on the friction, there exists exactly one rectilinear solution if the two points lie on the same ray, and at least two solutions traveling in opposite directions otherwise.

Piotr Zgliczynski (Jagiellonian University, Krakov)

Central configurations - some rigorous computer assisted result

I will give an overview of our of recent computer assisted proofs for the rigorous count of central configurations.

Our approach is based on: - the use of interval arithmetics methods, for example the Newton-Krawczyk operator - a priori bounds for central configurations

This allows to obtain an rigorous listing of all central configurations when masses are away from zero and there are no bifurcation nearby in the mass space, we have done for equal masses in the planar case for $n = 5, 6, 7$ and in the spatial case for $n = 5, 6$.

To extend this approach to all masses the following issues has to be solved: - understanding of restricted N+k problems (N-big masses and k "massless" bodies) and their continuation to full problem - the rigorous analysis of bifurcations

This is joint work with Małgorzata Moczurad.

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