INdAM Workshop - New Trends in Control Theory and PDEs

On the occasion of the 60th birthday of Piermarco Cannarsa

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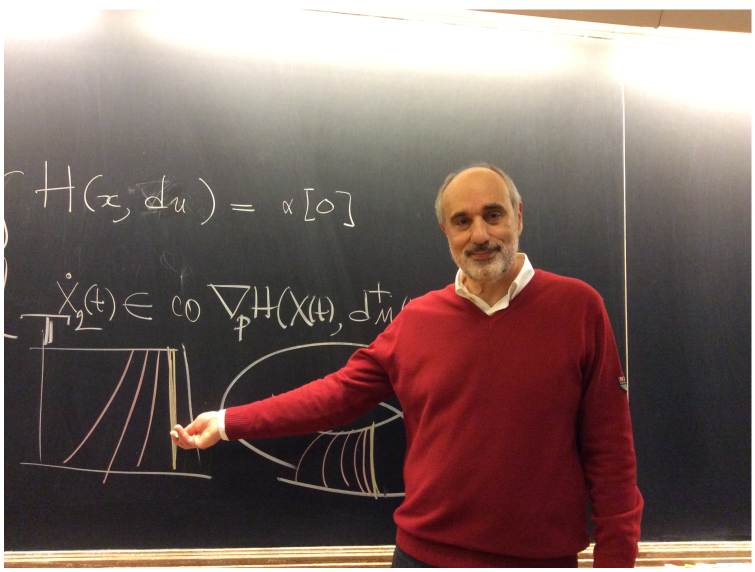
Book of Abstracts



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On the solution of the Dirichlet problem for the subelliptic eikonal equation

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Let $\Omega \subset \mathbb{R}^n$ be an open bounded set with smooth boundary, Γ, and let

$$X_1, \ldots, X_N, \quad (N \geq 2),$$

be a family of smooth (real) vector fields satisfying the Hörmander bracket generating condition. We consider the viscosity solution of the Dirichlet problem

$$\begin{cases} \sum_{j=1}^{N} (X_j T)^2(x) = 1, & x \in \Omega, \\ T(x) = 0, & x \in \Gamma\,. \end{cases} \tag{1}$$

The existence and the uniqueness of the viscosity solution of Equation (1) is well-known.

We study the regularity of T. We provide the following characterization: T fails to be Lipschitz continuous if and only if the minimum time problem naturally associated with Equation (1) admits the so called singular time-optimal trajectories. Furthermore, we give some sufficient conditions guaranteeing the absence of singular time-optimal trajectories.

References

[1] Paolo Albano, On the eikonal equation for degenerate elliptic operators, *Proc. Amer. Math. Soc.*, **140** (2012), pp. 1739–1747.

[2] Paolo Albano, Piermarco Cannarsa and Teresa Scarinci, Regularity results for the minimum time function with Hörmander vector fields, preprint (2017).

[3] Piermarco Cannarsa and Carlo Sinestrari, Convexity properties of the minimum time function, *Calc. Var. Partial Differential Equations*, **3** (1995), no. 3, pp. 273–298.

[4] Maklouf Derridj, Sur un théorème de traces, *Ann. Inst. Fourier (Grenoble)*, **22** (1972), no. 2, 73–83.

Joint work with: Piermarco Cannarsa (*Università di Roma "Tor Vergata"*), Teresa Scarinci (*University of Vienna*)

Uniqueness and non-uniqueness in Mean-Field Games systems of PDEs

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I consider backward-forward parabolic system of PDEs in $(0,T)\times\mathbb{R}^d$ associated to Mean Field Games with finite horizon T

$$\begin{cases} -v_t + H(Dv) = \frac{1}{2}\sigma^2(x)\Delta v + F(x, m(t,\cdot)), & v(T,x) = G(x, m(T)), \\ m_t - \operatorname{div}(DH(Dv)m) = \frac{1}{2}\Delta(\sigma^2(x)m), & m(0,x) = \nu(x), \end{cases}$$

in the unknowns (v,m), where $m(t,\cdot)$ is a probability density for each t, and the given running and terminal costs F, G map a subset of $\mathbb{R}^d\times\mathcal{P}(\mathbb{R}^d)$ into $\mathbb{R}$.

I'll first review the main conditions introduced by Lasry and Lions [4] for uniqueness of classical solutions, namely, convexity of H and monotonicity in m of the costs F, G.

In a lecture of 2009 [5] Lions showed that uniqueness can also be obtained by choosing instead T small (and H smooth). I'll present some uniqueness results for problems with small data inspired by that lecture (without convexity or monotonicity conditions) for either the problem above [3], or for systems with several populations and Neumann boundary conditions [2] motivated by the models of segregation in urban settlements of [1].

Finally, I'll show a class of evolutive Mean Field Games with multiple solutions for all time horizons T and convex but non-smooth Hamiltonian H, as well as for smooth H and T large enough. These examples of nonuniqueness from [3] show that the sufficient conditions described before for uniqueness are not far from being sharp.

References

[1] Y. Achdou, M. Bardi, M. Cirant: Mean Field Games models of segregation, Math. Models Methods Appl. Sci. 27 (2017), 75–113,

[2] M. Bardi and M. Cirant, to appear.

[3] M. Bardi and M. Fischer, On non-uniqueness and uniqueness of solutions in finite-horizon Mean Field Games, preprint 2017.

[4] J.-M. Lasry, P.-L. Lions: *Mean field games*, Jpn. J. Math. **2** (2007), 229–260.

[5] P.-L. Lions: Lectures at Collège de France 2008-9.

Joint work with: Markus Fischer and Marco Cirant (*Dipartimento di Matematica "Tullio Levi Civita", Università di Padova*),

PDE models of controlled growth

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Living tissues, such as stems, leaves and flowers in plants, or bones in animals, grow into a great variety of shapes. In some cases, Nature has found ways to control this growth with remarkable accuracy.

This talk will be concerned with some PDE models desribing modeling controlled growth, namely:

(I) Growth of 1-dimensional curves in R^3 (tree stems, vines), where the shape is determined by a feedback response to gravity and to external obstacles.

(II) Growth of 2 or 3-dimensional domains, controlled by the concentration of a morphogen, coupled with the minimization of an elastic deformation energy.

Some recent existence, uniqueness, and stability results will be presented, together with numerical simulations. Further directions will be discussed.

References

[1] A. Bressan and M. Lewicka, A model of controlled growth, submitted.

[2] A. Bressan, M. Palladino, and W. Shen. Growth models for tree stems and vines, *J. Differential Equations*, to appear.

[3] A. Bressan and M. Palladino, Well-posedness of a model for the growth of tree stems and vines, submitted.

[4] F. Ancona, A. Bressan, O. Glass, and W. Shen, Feedback stabilization of stem growth, in preparation.

Joint work with: Fabio Ancona (*U. of Padova*) Olivier Glass (*Ceremade, U. Paris-Dauphine*), Marta Lewicka (*U. of Pittsburgh*), Michele Palladino (*Penn State U.*), and Wen Shen (*Penn State U.*)

Stochastic and worst-case shape optimization problems

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We consider shape optimization problems in which the data are not completely identified. A first case is when the right-hand side in the state equation is only known up to a probability P in the space $L^2(D)$; another case is when, under some uncertainty on the right-hand side, the worst situation is optimized. The most difficult situation occurs when the data in the cost functional and in the state equation may change sign, and so no monotonicity assumption is satisfied. Nevertheless, we are able to prove that an optimal domain exists.

References

[1] J.C. Bellido, G. Buttazzo, B. Velichkov: *Worst-case shape optimization for the Dirichlet energy.* Nonlinear Anal., **153** (2017), 117–129, DOI: 10.1016/j.na.2016.05.014.

[2] D. Bucur, G. Buttazzo: *Variational Methods in Shape Optimization Problems.* Progress in Nonlinear Differential Equations **65**, Birkhäuser Verlag, Basel (2005).

[3] G. Buttazzo, G. Dal Maso: *An existence result for a class of shape optimization problems.* Arch. Rational Mech. Anal., **122** (1993), 183–195.

[4] G. Buttazzo,, B. Velichkov: *A shape optimal control problem and its probabilistic counterpart.* Paper in preparation.

A few results on the weak maximum principle for elliptic equations

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I will present a few more or less recent results about the validity of the weak Maximum Principle for fully nonlinear degenerate elliptic operators.
The following topics will be discussed:
- validity of the weak Maximum Principle and positivity of a generalized principal eigenvalue,
- one directional degenerate elliptic operators in special unbounded domains,
- Phragmen-Lindelöf type results.

References

[1] H. Berestycki, I. Capuzzo Dolcetta, A. Porretta, L. Rossi, Maximum principle and generalized principal eigenvalue for degenerate elliptic operators. *J. Math. Pures Appl.,* (2015) Volume 9 no.5 , pp. 1276-1293

[2] I. Capuzzo Dolcetta, F. Leoni, A. Vitolo, The Alexandrov-Bakelman-Pucci weak Maximum Principle for fully nonlinear equations in unbounded domains. *Comm. Partial Differential Equations* 30 (2005), no. 10-12, pp. 1863-1881

[3] I. Capuzzo Dolcetta, A. Vitolo, The weak Maximum Principle for degenerate elliptic operators in unbounded domains, *International Mathematics Research Notices*, Vol. 2016, (2016)

Stable solutions in potential mean field game systems

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We introduce the notion of stable solution in mean field game theory: they are locally isolated solutions of the mean field game system. We prove that such solutions exist in potential mean field games and are local attractors for learning procedures.

Joint work with Ariela Briani (*U. Tours*)

On the singular dynamics of the viscosity solutions

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Let H be a Tonelli Hamiltonian. We consider the propagation of singularities along generalized characteristics by an intrinsic method. We will show that, for a prescribed solution u which has the representation in the form of inf-convolution, the relevant precess of sup-convolution determines the propagation of singulars and generalized characteristics for singular initial data. This method leads to the global result under mild Tonelli conditions.

Based on this global result, we can discuss the associated singular dynamics in both topological and differential sense. We obtained the homotopy equivalence result between the complement of Aubry set and the cut locus of u, and the local path-connected result of the cut locus. We also studied the ω-limit set of the semi-flows defined by generalized characteristics, and their connections to the regular dynamics.

References

[1] Cannarsa, Piermarco; Cheng, Wei; Fathi, Albert; *On the topology of the set of singularities of a solution to the HamiltonJacobi equation.* C. R. Math. Acad. Sci. Paris **355** (2017), no. 2, 176–180.

[2] Cannarsa, P.; Cheng, W., *Generalized characteristics and Lax-Oleinik operators: global result*, preprint, arXiv:1605.07581, 2016.

[3] Cannarsa, Piermarco; Cheng, Wei; Zhang, Qi, *Propagation of singularities for weak KAM solutions and barrier functions.* Comm. Math. Phys. 331 (2014), no. 1, 1–20.

Joint work with: Piermarco Cannarsa (*Dipartimento di Matematica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, 00133 Roma, Italy*), Albert Fathi (*ENS de Lyon & IUF, UMPA, 46 Allée d'Italie, 69007 Lyon, France*)

Error and fallacy in control

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The talk examines the various types of mistakes that occur in both optimal and stabilizing control.

Moreau's sweeping process and its optimal control

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The sweeping process (*processus de rafle*) is the rate independent evolution inclusion

$$\dot{x}(t) \in -N_{C(t)}(x(t)) \ \ \text{(with the initial condition } x(0) = x_0 \in C(0))$$

that was introduced by Moreau in the Seventies as a tool for modeling unilateral constraints in a quasi-static framework. Here $C(t)$ is a moving mildly nonconvex set, whose t-dependence may be Lipschitz or even BV, and $N_{C(t)}(x)$ is the normal cone to C at $x \in C$. This problem was studied from the viewpoint of existence of solutions by several authors, mainly around Montpellier's school (Moreau, Castaing, Valadier, Thibault, Monteiro Marques, Bounkhel, ...). The optimal control of its perturbed version

$$\dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)), u(t) \in U$$

is much less studied, although similar topics are gaining some interest in different contexts, including variational inequalities and rate independent processes. Since the condition $x(t) \in C(t)$ for all t is built in the dynamics, this may be seen as a different approach to model state constraints. I will describe some results appearing in the papers quoted below, focusing mainly on necessary optimality conditions.

References

[1] Ch. E. Arroud, G. Colombo, A Pontryagin maximum principle for the controlled sweeping process, *Set-Val. Var. Anal.*, in print, DOI 10.1007/s11228-017-0400-4

[2] Ch. E. Arroud, G. Colombo, Necessary conditions for a nonclassical control problem with state constraints, *Proceedings of IFAC 2017 World Congress*, Toulouse, July 9–14, 2017.

[3] G. Colombo, M. Palladino, The minimum time function for the controlled Moreau's sweeping process, *SIAM J. Control* 54 (2016), 2036-2062.

[4] G. Colombo, R. Henrion, Nguyen D. Hoang, B. S. Mordukhovich, Optimal control of the sweeping process: the polyhedral case, *J. Differential Eqs.* 260 (2016), 3397-3447.

On the small-time global controllability of the Navier-Stokes equations with the Navier-slip boundary condition

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We prove the global controllability of the Navier-Stokes condition when the fluid slips on the boundary according to the Navier condition. The proofs rely on the following ingredients: the return method (Coron), the controllability of the Euler equations of incompressible fluids (Coron and Glass), asymptotic boundary layer expansion (Iftimie and Sueur), dissipation of the boundary layer (Marbach), and a local null controllability result (Fernandez-Cara, Guerrero, Imanuvilov and Puel for the Stokes no-slip boundary condition, extended by Guerrero to the case of the Navier slip boundary condition).

Joint work with: Frédéric Marbach (*UPMC, LJLL*), Franck Sueur (*Université de Bordeaux, IMB*

Invariant sets for solutions of Stochastic Partial Differential Equations

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Let H be a separable Hilbert space. We are concerned with stochastic differential equations of the form

$$\begin{cases} dX = (AX + B(X))dt + \sigma(X)dW(t), \\ \\ X(0) = x \in H, \end{cases} \tag{1}$$

where $A : D(A) \subset H \to H$ is the infinitesimal generator of a strongly continuous semigroup in H, $B : D(A) \subset H \to H$ is nonlinear, $\sigma : H \to L_2(H)$ ($L_2(H)$ is the space of all Hilbert–Schmidt operators in H) and $W(t),\ t \geq 0$, is a cylindrical Wiener process on H. We shall assume that problem (1) has a unique solution $X(\cdot, x)$ for all $x \in H$.

Let K be a closed subset of H. Then we say that K is *invariant* if $X(t, x) \in K$ for all $x \in H$ and all $t > 0$.

We shall present necessary and sufficient conditions, obtained in collaboration with Piermarco Cannarsa and Hélène Frankowska, for the invariance property of several SPDEs.

Multiplicative controllability for semilinear reaction-diffusion equations

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In this talk we present some results concerning the global approximate controllability of semilinear reaction-diffusion equations governed via the coefficient of the reaction term (multiplicative or bilinear control). We study both uniformly parabolic and degenerate equations.

We start to consider a one-dimensional uniformly parabolic problem and we extend in [1] some nonnegative controllability results by bilinear controls assuming that both the initial and target states admit no more than finitely many changes of sign. Our method uses a technique, introduced in [1], employing the shifting of the points of sign change by making use of a finite sequence of initial-value pure diffusion problems.

The results obtained in [1] have allowed us to approach in [2] the multidimensional case with radial symmetry. Indeed, assuming that the initial and final data are radial on a ball about the origin, we reduce the problem to a one-dimensional setting. In this way, we obtain an approximate controllability result for data which admit finitely many hyperspheres of sign change.

The method, introduced in [1] for uniformly parabolic equations, can be also extended to degenerate reaction-diffusion equations (see [3]). Our interest in degenerate parabolic equations is motivated by the study of mathematical models for some energy balance models in climatology, in particular for the Budyko-Sellers model.

References

[1] P. Cannarsa, G. Floridia, A.Y. Khapalov, Multiplicative controllability for semilinear reaction-diffusion equations with finitely many changes of sign, preprint (2016), to appear on *Journal de Mathématiques Pures et Appliquées.*

[2] G. Floridia, Multiplicative controllability for multidimensional reaction-diffusion equations, preprint (2017).

[3] G. Floridia, C. Nitsch, C. Trombetti, Controllability of nonlinear degenerate parabolic equations governed by multiplicative control in the reaction term, preprint (2017).

Maximum Principle and Sensitivity Relations for the Infinite Horizon Optimal Control Problem

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Consider the infinite horizon optimal control problem

$$V(t_0, x_0) := \inf \int_{t_0}^{\infty} L(t, x(t), u(t))\, dt \tag{2}$$

over all trajectory-control pairs (x, u), subject to the state equation

$$\begin{cases} x'(t) = f(t, x(t), u(t)), \quad u(t) \in U(t) \quad \text{for a.e. } t \geq 0 \\ x(t_0) = x_0. \end{cases} \tag{3}$$

It is well known that under mild assumptions every optimal solution satisfies a, possibly abnormal, maximum principle, not involving transversality conditions. To investigate normality as well as transversality conditions, our main results exploit the value function V. For instance, we show that if $V(0, \cdot)$ is merely lower semicontinuous, then for a dense subset of initial conditions a normal maximum principle, augmented by sensitivity relations involving the Fréchet subdifferentials of $V(t, \cdot)$, holds true. We also prove that for every upper semicontinuous function Φ "supporting" locally $V(t_0, \cdot)$ at x_0 a maximum principle can be stated with the transversality conditions involving limiting (normal case) or horizontal limiting (abnormal case) supegradients of Φ at x_0. This yields normality of the maximum principle for calm problems. Furthermore, if V is locally Lipschitz, then a normal maximum principle together with sensitivity relations involving generalized gradients of V holds true. Such relations simplify drastically the investigation of the limiting behaviour at infinity of the adjoint state. Finally, we discuss some issues related to second order sensitivity relations.

References

[1] P. Cannarsa, H. Frankowska, Some characterizations of optimal trajectories in control theory, *SIAM J. Control Optim.*, **Volume no. 29** (1991), pp. 1322-1347.

[2] P. Cannarsa, H. Frankowska, From pointwise to local regularity for solutions of Hamilton-Jacobi equations, *Calculus of Variations and PDEs*, **Volume no. 49** (2013), pp. 1061-1074.

[3] P. Cannarsa, H. Frankowska & T. Scarinci, Second-order sensitivity relations and regularity of the value function for Mayer's problem in optimal control, *SIAM J. Control Optim.*, **Volume no. 53** (2016), pp. 3642-3672.

[4] P. Cannarsa, H. Frankowska, Value function, relaxation, and transversality conditions in infinite horizon optimal control, to appear in *JMAA*.

Joint work with: Piermarco Cannarsa (*Università di Roma "Tor Vergata"*)

Controllability properties of degenerate parabolic equations

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Consider the one-dimensional degenerate parabolic equation

$$\begin{cases} y_t - (a(x)y_x)_x = 0\,, & \text{in } \; Q := (0,1) \times (0,T)\,, \\ y(0,t) = u(t)\,, & t \in (0,T)\,, \\ y(1,t) = 0\,, & t \in (0,T)\,, \\ y(x,0) = y_0(x)\,, & x \in (0,1)\,, \end{cases} \tag{4}$$

where the diffusion coefficient $a : [0,1] \to \mathbb{R}$ is degenerate at $x = 0$, i.e., $a(0) = 0$ and $a > 0$ otherwise, and the control function u acts through the boundary at the degeneracy point $x = 0$. We focus on the weakly degenerate case, that is, we assume the diffusion coefficient to satisfy the relation $\limsup_{x\to 0^+} \frac{xa'(x)}{a(x)} = K$ for some $K < 1$. Indeed, without this assumption, the solution y to (4) may well not admit trace at $x = 0$. We prove the approximate controllability of equation (4) with control functions $u \in H^1_0(0,T)$, relying on the duality argument that allows to recast the controllability problem in terms of unique continuation properties of the adjoint operator

$$L^* v = v_t + (av_x)_x\,.$$

For this reason, we establish suitable local Carleman estimates with a spatial weight function tailored to the degenerate diffusion coefficient.
Moreover, we apply the previous result to prove approximate controllability of a weakly degenerate parabolic equation with interior degeneracy (for example, in the case of $x \in (-1,1)$ and $a(0) = 0$) and control function supported on one side only of the degeneracy point (for example, $supp(u) \subset (-1,0)$), by exploiting the transmission of information across the degeneracy.

References

[1] Cannarsa, Piermarco, and Guglielmi, Roberto, Approximate controllability of a class of degenerate parabolic equations, *submitted* (2017), pp. 1-20.

[2] Cannarsa, Piermarco, and Tort, Jacques, and Yamamoto, Masahiro, Unique continuation and approximate controllability for a degenerate parabolic equation, *Applicable Analysis*, Taylor & Francis (2012), 91 (8), pp. 1409-1425.

Joint work with: Piermarco Cannarsa (*Università di Roma Tor Vergata, Italy).*

Control of PDEs including degeneration

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We consider different mathematical models that exhibit degeneration such as the linear wave equation with degenerating stiffness, the weighted p-Laplace problem, where the weight can be seen as a control. Here degeneration can take place in the control and due to the properties of the p-Laplace for $p > 2$. We also provide modeling of dynamic damage evolution and consider optimal controls in this context. For the $1 - d$ wave equation we show lack of observability for a critical exponent of the degenerating coefficient. This part is joint work with P. Cannarsa and F. Alabau. As for the p-Laplcean, we report on some recent work with E. Casas and P. Kogut.

Kuramoto-Lohe Synchronization over quantum networks

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We investigate a non-Abelian generalization of the Kuramoto model proposed by Lohe and given by N quantum oscillators (nodes) connected by a quantum network where the wave-function at each node is distributed over quantum channels to all other connected nodes. It leads to a system of Schrödinger equations coupled by nonlinear self-interacting potentials given by their correlations.

We give a complete picture of synchronization results, given on the relative size of the natural frequency and the coupling constant, for two non-identical oscillators and show complete phase synchronization for arbitrary $N > 2$ identical oscillators. Our results are mainly based on the analysis of the ODE system satisfied by the correlations and on the introduction of a quantum order parameter, which is analogous to the one defined by Kuramoto in the classical model. As a consequence of the previous results, we obtain the synchronization of the probability and the current densities defined via the Madelung transformations.

We also discuss the Wigner-Lohe model for quantum synchronization which can be derived from the Schrödinger-Lohe model using the Wigner formalism. For identical one-body potentials, we provide a priori sufficient framework leading the complete synchronization, in which L^2-distances between all wave functions tend to zero asymptotically.

References

[1] Paolo Antonelli, Pierangelo Marcati, A model of synchronization over quantum networks, arXiv:1702.00041, to appear on *Journal of Physics A: Mathematical and Theoretical*.

[2] Paolo Antonelli, Seung-Yeal Ha, Dohyun Kim, Pierangelo Marcati, The Wigner-Lohe model for quantum synchronization and its emergent dynamics, arXiv:1702.03835, to appear on *Networks and Heterogeneous Media*

Joint work with: Paolo Antonelli (*Gran Sasso Science Institute GSSI*)

The cost of controlling degenerate parabolic equations

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We consider the typical one-dimensional degenerate parabolic equation

$$\begin{cases} u_t - (x^\alpha u_x)_x = h(x,t)\chi_{[a,b]}(x) & x \in (0,1),\ t > 0, \\ (x^\alpha u_x)(0,t) = 0, & t > 0, \\ u(1,t) = 0, & t > 0, \\ u(x,0) = u_0(x), & x \in (0,1), \end{cases} \tag{5}$$

for which null controllability holds when $\alpha \in [0,2)$. The aim of this work is to understand the behavior of the cost of control as $\alpha \to 2^-$ (since $\alpha = 2$ is the threshold for null controllability), and/or $T \to 0^+$ (the 'fast control problem').

We prove that the null controllability cost, defined as

$$C_{ctr}(\alpha, T) := \sup_{\|u_0\|=1} \inf\{\|h\|_{L^2((0,1)\times((0,T))}, u^{(h)}(T) = 0\},$$

blows up as $\alpha \to 2^-$, and/or as $T \to 0^+$, at a precise speed:

$$e^{-\frac{1}{C}\frac{1}{(2-\alpha)^{4/3}}(\ln\frac{1}{2-\alpha}+\ln\frac{1}{T})} e^{\frac{C}{T(2-\alpha)^2}} \leq C_{ctr}(\alpha, T) \leq e^{\frac{C'}{T(2-\alpha)^2}}.$$

Our analysis builds on the moment method, developed by Fattorini-Russel, Seidman, Güichal, Tenenbaum-Tucsnak and Lissy, precising some general existence results for suitable biorthogonal families under some 'asymptotic gap conditions', and obtaining some new fine properties of the Bessel functions J_ν of large order ν (obtained by ordinary differential equations techniques).

References

[1] P. Cannarsa, P. Martinez, J. Vancostenoble, The cost of controlling weakly degenerate parabolic equations by boundary controls, *Math. Control Relat. Fields*, **Volume no.2** (2017), pp. 171-211

[2] P. Cannarsa, P. Martinez, J. Vancostenoble, Precise estimates for biorthogonal families under asymptotic gap conditions (2017)

[3] P. Cannarsa, P. Martinez, J. Vancostenoble, The cost of controlling strongly degenerate parabolic equations (2017)

Joint work with: Piermarco Cannarsa (*Dipartimento di Matematica, Università di Roma "Tor Vergata"*), and Judith Vancostenoble (*Institut de Mathématiques de Toulouse, UMR CNRS 5219, Université Paul Sabatier Toulouse III*)

Necessary optimality conditions for infinite dimensional state constrained control problems

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We consider a Mayer problem associated to a semilinear control system of the form

$$\dot{x}(t) = Ax(t) + f(t, x(t), u(t)),$$

with $x(\cdot)$ staying in a given closed subset of an infinite dimensional separable Banach space. The presence of the operator A, the infinitesimal generator of a strongly continuous semigroup, makes this system a convenient tool for the study of control problems involving PDEs. We provide estimates on the distance of a mild solution of the control system to the set of all solutions lying in the given constraint set. These results can be applied to derive necessary optimality conditions.

Joint work with: Hélène Frankowska (*CNRS, IMJ-PRG, Paris 6*), Elsa Maria Marchini (*Dipartimento di Matematica "F. Brioschi", Politecnico di Milano*)

Energy decay estimates for abstract evolution equations with time delay

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We consider abstract semilinear evolution equations with a time lag and assume that the C_0-semigroup associated to the linear (undelayed) part of the model is exponentially stable. Then, under some Lipschitz continuity assumptions on the nonlinear term, we will show that the whole system retains this property when appropriate smallness conditions on the time delay or on the initial data are satisfied. Some concrete applications of the abstract arguments are also illustrated.

Joint work with: Serge Nicaise (*Université de Valenciennes*)

Mayer Control Problem with Probabilistic Uncertainty on Initial Positions and Velocities

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We consider the following controlled differential equations

$$\dot{x}(t) = f(x(t), u(t)),\ u(t) \in U,\ t \in [0, T], \tag{6}$$

where $f : \mathbb{R}^d \times U \mapsto \mathbb{R}^d$ is a Lipschitz function, U is a compact subset of some finite dimensional vector space. The controller aims consists in minimizing the cost

$$J := g(x(T))$$

over all trajectories to (6) where $g : \mathbb{R}^d \mapsto \mathbb{R}$ is a bounded Lipschitz function.

The main specificities of the optimal control problem we will investigate in the paper lies in both following facts

- The initial position is not exactly known by the controller but only a probability measure μ_0 is available.

- Because of the uncertain initial position at every point of the support of μ_0 corresponds a possible different control. Moreover we allow the "division of mass" i.e. even the initial condition x_0 would be known (namely $\mu_0 = \delta_{x_0}$), it can be splitted in different trajectories by several possible velocities in $f(x_0, U)$ but of course the weight of these trajectories should be remain 1.

So the natural state variable of our control problem is probability measure on $\mathbb{R}^d$. The conservation of mass of the trajectory $(\mu_t)_{t\in[0,T]}$ and the controlled dynamics can be summarized in the following dynamical system

$$\begin{cases} \partial_t \mu_t + \operatorname{div}(v_t \mu_t) = 0, & t \in [0, T] \\ \mu|_{t=0} = \mu_0 & \\ v_t(x) \in f(x, U) & \text{for } \mu_t \text{ almost every } x \in \mathbb{R}^d \ . \end{cases}$$

The first equation of the above system should be understood in the sense of distribution.

So the new state variable is a probability measure, and our aim is to characterize the associate value function through a Hamilton Jacobi Bellman Equation state on the space probability measures on $\mathbb{R}^d$.

References

[1] Cardaliaguet P. & Quincampoix M. , Deterministic differential games under probability knowledge of initial condition , *Int. Game Theory Rev.*, **Volume no. 10-1** (2008), pp. 1-16.

[2] Marigonda A. & Quincampoix M. , Mayer Control Problem with Probabilistic Uncertainty on Initial Positions and Velocities , *In Preparation*,

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Joint work with: Antonio Marigonda (*Department of Computer Science, University of Verona, Italy.*)

The Sard conjecture on Martinet surfaces

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Given a totally nonholonomic distribution of rank two on a three-dimensional manifold, we will address the size of the set of points that can be reached by singular horizontal paths starting from a same point. In this setting, the Sard conjecture states that that set should be a subset of the so-called Martinet surface of 2-dimensional Hausdorff measure zero. We will show how to prove the conjecture in the case where the Martinet surface is smooth. Then, we will explain how the result can be extended to some class of singular real-analytic Martinet surfaces satisfying a non-transversality condition. The methods rely on control theory, dynamics, and some techniques of resolution of singularities. This is a joint work with Andr Belotto.

References

[3] A. Belotto da Silva, L. Rifford, The Sard conjecture on Martinet surfaces, preprint (2016).

Joint work with: André Belotto da Silva (*Université Paul Sabatier-Toulouse III*).

Bi-metric regularity properties and time-discretization schemes for LQ problems with bang-bang solutions

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In many applications the control variable is constrained and appears linearly in the dynamic system and in the cost functional. Therefore, the optimal controls exhibit in general bang-bang arcs. While coercive problems are well studied and understood in the literature, less results address the aforementioned problem.

In this talk we are interested in the investigation of the stability of linear control problems under perturbations. We focus our attention on linear-quadratic problems where controls are constrained to take values in the m-dimensional hypercube $[-1, 1]^m$. The optimal controls will be supposed to be purely bang-bang. The stability properties of the above problem will be investigated and described via the notion of bi-metric regularity [1]. Furthermore we see how the theory of metric regularity can be applied in infinite dimension to study the convergence of numerical approximations. More precisely, we introduce a numerical time-discretization scheme satisfying a high order of convergence [2].

References

[1] J. Preininger, T. Scarinci, V. Veliov, On the regularity of linear-quadratic optimal control problems with bang-bang solutions. (Preprint, 2017)

[2] T. Scarinci, V. Veliov. Higher-order numerical scheme for linear-quadratic problems with bang-bang controls. (Preprint, 2017)

Partly joint work with: Vladimir Veliov and Jakob Preininger (*Vienna University of Technology*), and Marc Quinquampoix (*University of Western Brittany*).

Martingale Optimal Transport

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The original transport problem is to optimally move a pile of soil to an excavation. Mathematically, given two measures of equal mass, we look for an optimal map that takes one measure to the other one and also minimizes a given cost functional. Kantorovich relaxed this problem by considering a measure whose marginals agree with given two measures instead of a bijection. This generalization linearizes the problem. Hence, allows for an easy existence result and enables one to identify its convex dual.

In robust hedging problems, we are also given two measures. Namely, the initial and the final distributions of a stock process. We then construct an optimal connection. In general, however, the cost functional depends on the whole path of this connection and not simply on the final value. Hence, one needs to consider processes instead of simply the transport maps. The probability distribution of this process has prescribed marginals at final and initial times. Thus, it is in direct analogy with the Kantorovich measure. But, financial considerations restrict the process to be a martingale. Interestingly, the dual also has a financial interpretation as a robust hedging (super-replication) problem.

In this talk, we prove an analogue of Kantorovich duality: the minimal super- replication cost in the robust setting is given as the supremum of the expectations of the contingent claim over all martingale measures with a given marginal at the maturity.

References

[1] Y. Dolinsky and H.M. Soner, Robust Hedging and Martingale Optimal Transport in Continuous Time, *Probability Theory and Related Fields*, **160(1-2)**, (2014), 391–427.

[2] Y. Dolinsky and H.M. Soner, Martingale optimal transport in the Skorokhod space, *Stochastic Processes and Applications*, **125(10)**, (2015), 3893–3931.

[3] I. Ekren and H.M. Soner, Constrained optimal transport, preprint.

[4] M. Burzoni, F. Riedel and H. M. Soner, Viability, Arbitrage and Preferences, preprint.

Joint work with: Yan Dolinsky (*Hebrew University*), Ibrahim Ekren (*ETH Zurich*), Matteo Burzoni (*ETH Zurich*), Frank Riedel (*University of Bielefeld*).

Observability inequalities on measurable sets for the Stokes system

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In this paper, we establish spectral inequalities on measurable sets of positive Lebesgue measure for the Stokes operator, as well as observability inequalities on space-time measurable sets of positive measure for non-stationary Stokes system.

Joint work with: Felipe W. Chaves-Silva (*Department of Mathematics, Federal University of Pernambuco*), Can Zhang (*School of Mathematics and Statistics, Wuhan University*)

Observability and controllability for waves

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I will describe some results obtained recently about waves with internal observation or control domain:

- Observability or controllability of waves with a time-varying domain (with Jérôme Le Rousseau, Gilles Lebeau, Peppino Terpolilli), and I will illustrate what we call the time-GCC on several examples.
- Estimate of the observability constant in large time (with Emmanuel Humbert and Yannick Privat): $\frac{C_T}{T}$ converges to the minimum of a spectral and a geometric quantity. I will present some consequences on characterizing Zoll manifolds, and on con trollability of Schrödinger.

References

[1] J. Le Rousseau, G. Lebeau, P. Terpolilli, E. Trélat, Geometric control condition for the wave equation with a time-dependent domain, *Anal. PDE*, **10 no. 4** (2017), pp. 983-1015

[2] E. Humbert, Y. Privat, E. Trélat, Observability properties of the homogeneous wave equation on a closed manifold, preprint (2016).

Joint works with:
J. Le Rousseau (Univ. Orléans), G. Lebeau (Univ. Nice), P. Terpolilli (Total),
E. Humbert (Univ. Tours), Y. Privat (CNRS, Univ. Paris 6).

Fully convex control problems with state constraints and impulses

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A Fully Convex Control (FCC) problem has the appearance of the classical calculus of variations Bolza problem

$$\min \int_0^T L(x(t), \dot{x}(t))dt + \ell(x(0), x(T)),$$

where the minimization is over $x(\cdot)$ belonging to some class of arcs. The distinguishing features of FCC are that the data $L(\cdot,\cdot)$ and $\ell(\cdot,\cdot)$ (i) may take on the value $+\infty$ and (ii) are convex functions. Allowance of (i) provides great flexibility incorporating constraints so that most standard control problems come under its purlieu. However, broad generality is restrained by (ii), which although quite special, nonetheless includes the classical linear quadratic regulator and many of its generalizations. Moreover, the speciality of (ii) opens up the possibility of using convex dual formulations.

We shall review the Hamilton-Jacobi (HJ) theory for FCC problems when the data has no implicit state constraints and is coercive, in which case the minimizing class of arcs are Absolutely Continuous (AC). When a state constraint $x(t) \in X$ is added to the problem formulation, the dual variable is likely to exhibit an impulse or "jump" when the constraint is active. The two properties of a state constraint and allowing impulsive arcs are in fact dual to each other, and the minimizing class becomes those of bounded variation. We shall describe Rockafellar's optimality conditions for these problems and a new technique for approximating them by AC problems that utilizes Goebel's self-dual envelope. The approximating AC problems maintain duality and the existing theory can be applied to them. It is proposed that an HJ theory can be developed for BV problems as an appropriate limit of the approximating AC problems. An explicit example will be provided to illustrate this.

Joint work with Cristopher Hermosilla (Louisiana State University).

Carleman estimates for viscoelasticity equations and applications

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We consider equations of viscoelasticity. One of them is described as follows.

$$\rho(x)\partial_t^2 \mathbf{y}(x,t) = L_{\lambda,\mu}\mathbf{y}(x,t) - \int_0^t L_{\widetilde{\lambda},\widetilde{\mu}}\mathbf{y}(x,\eta)d\eta + \mathbf{F}(x,t) \quad \text{in } \Omega \times (0,T) \qquad (*)$$

with $\mathbf{y} = 0$ on $\partial\Omega \times (0,T)$. Here $\Omega \subset \mathbf{R}^n$, $n = 2, 3$ is a smooth bounded domain, $\vec{\nu}$ is the unit outward normal vector to $\partial\Omega$, $\partial_k = \frac{\partial}{\partial x_k}$, $\nabla = (\partial_1, ..., \partial_n)$, $\nabla \mathbf{y} = (\partial_j y_k)_{1\le j,k\le n}$

and $\varepsilon_{jk}(\mathbf{y}) = \frac{1}{2}(\partial_j y_k + \partial_k y_j)$ for $\mathbf{y} = (y_1, ..., y_n)^T$, $\cdot^T$ denotes the transpose, and we set $L_{\lambda,\mu}\mathbf{u}(x,t) = \mu(x)\Delta\mathbf{u} + (\lambda(x)+\mu(x))\nabla \mathrm{div}\,\mathbf{u} + (\mathrm{div}\,\mathbf{u})\nabla\lambda + (\nabla\mathbf{u} + (\nabla\mathbf{u})^T)\nabla\mu$ and $L_{\widetilde{\lambda},\widetilde{\mu}}\mathbf{u}(x,\eta) = \widetilde{\mu}(x,t,\eta)\Delta\mathbf{u} + (\widetilde{\lambda}(x,t,\eta) + \widetilde{\mu}(x,t,\eta))\nabla\mathrm{div}\,\mathbf{u} + (\mathrm{div}\,\mathbf{u})\nabla\widetilde{\lambda} + (\nabla\mathbf{u} + (\nabla\mathbf{u})^T)\nabla\widetilde{\mu}$.

Let $\Gamma \subset \partial\Omega$ be a suitable subboundary and let μ, λ satisfy appropriate conditions such as the pseudo-convexity. For the viscoelasticity equation, we discsuss

- **Observavbility inequality.**
  Let $\mathbf{F} = 0$ in (*). Then determine the energy
  $$\int_\Omega \left(\lambda(x)|\mathrm{div}\,\mathbf{y}(x,0)|^2 + 2\mu(x) \sum_{k,j=1}^{n} |\varepsilon_{jk}(\mathbf{y})(x,0)|^2 + \rho(x)|\partial_t \mathbf{y}(x,0)|^2 \right) dx$$
  by a suitable norm of $\partial_{\vec{\nu}}\mathbf{y}$ on $\Gamma \times (0,T)$.

- **Stability for an inverse source problem.**
  Let $\mathbf{y}(\cdot,0) = \partial_t\mathbf{y}(\cdot,0) = 0$ in Ω and $\mathbf{F}(x,t) = R(x,t)\mathbf{f}(x)$ where R is a known $n \times n$ matrix function and $\mathbf{f}$ is an $\mathbf{R}^n$-valued function. Then estimate $\|\mathbf{f}\|_{H^1(\Omega)}$ by a suitable norm of $\partial_{\vec{\nu}}\mathbf{y}$ on $\Gamma \times (0,T)$.

- **Stability for a coefficient inverse problem.**
  Let $\mathbf{F} = 0$ and let us choose $\mathbf{y}(x,0)$ and $\partial_t\mathbf{y}(x,0)$ suitably. In (*), we assume the following forms :$\widetilde{\lambda}(x,t,\eta) = \ell(x)p(t,\eta)$ $\widetilde{\mu}(x,t,\eta) = m(x)q(t,\eta)$ with given functions $p(t,\eta)$, $q(t,\eta)$. Then stably determine $\ell(x)$ and $m(x)$ by $\vec{\nu}\mathbf{y}$ on $\Gamma \times (0,T)$.

In the existing papers (e.g., [1, 4, 5]), data $\partial_{\vec{\nu}}\mathbf{y}$ must be taken on the whole lateral boundary $\partial\Omega \times (0,T)$ because of the difficulty in deriving a Carleman estimate for functions without compact supports. Here we first show a Carleman estimate for (*) for $\mathbf{y}$ not having compact supports and then apply the Carleman estimate to the above three problems. Our main results are the Lipschitz stability with sufficently large $T > 0$ and suitable conditions on λ, μ. Also we have to treat the integral term in (*) by a similar way to [2] and [3].

References

[1] M. de Buhan and A. Osses, *Logarithmic stability in determination of a 3D viscoelastic coefficient and a numerical example,* Inverse Problems **26** (2010), 095006.

[2] C. Cavaterra, A. Lorenzi and M. Yamamoto, *A stability result via Carleman estimates for an inverse source problem related to a hyperbolic integro-differential equation,* Comput. Appl. Math. **25** (2006), 229-250.

[3] P. Loreti, D. Sforza and M. Yamamoto, *Carleman estimate and application to an inverse source problem for a viscoelasticity model in anisotropic case*, preprint.

[4] A. Lorenzi, F. Messina and V.G. Romanov,*Recovering a Lamé kernel in a viscoelastic system,* Appl. Anal. **86** (2007), 1375-1395.

[5] A. Lorenzi and V.G. Romanov, *Stability estimates for an inverse problem related to viscoelastic media,* J. Inverse Ill-Posed Problems **18** (2006), 407-435.

Joint work with Oleg Y. Imanuvilov (*Colorado State University*).

PMC-EZ =2

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My PMC # (= Piermarco Cannarsa Number) is 2. For some reason it is not 1. It could have been 1, since we have had many subjects of common interest and we know each other for a long time. But it is 2.

My PMC # is 2 because Piermarco published a joint paper in collaboration with Vilmos Komornik. I did it as well, thirty years back. They worked together on a very interesting problem, related to the controllability of the semilinear wave equation, using sidewise methods. I will discuss this issue, presenting some open problems, and some other closely related questions arising in the numerical approximation of the minimal control time, a joint work in collaboration with Jérôme Lohéac (Nantes).

But we have had several other non-empty thematic intersections with Piermarco. I will also present some recent joint work in collaboration with Emmanuel Trélat and Jiamin Zhu (UMPC-Paris) on the optimal control of traveling wave solutions of reaction-diffusion problems arising in population dynamics, a topic which is closely related to a recent work of Piermarcos with Floridia and Khapalov.

The talk will avoid technical difficulties, being oriented to a wide audience.