ABSTRACTS WORKSHOP “OPTIMAL CONTROL PROBLEMS AND MEAN FIELD GAMES IN LIFE SCIENCES AND ECONOMICS”

Awerkin Almendra

(Università di Pavia)

“Optimal installation of renewable electricity sources with a large number of producers: a mean-field approach”

Abstract. In this work we study the problem of finding a Pareto optimum in a stochastic irreversible installation problem under a constrained total capacity θ in a market with N >> 1 firms (possibly going to $+\infty$). For a fixed and finite N, this problem is studied and solved in a more abstract situation in [1], where they state infinite dimensional stochastic Kuhn-Tucker conditions to prove optimality of policies. The optimal solution proposed there is as follows: every firm increases its investment until it reaches the critical level θ/N, whenever the initial condition is lower than this critical level. Nevertheless, when $N \to +\infty$, this strategy results to be not admissible as soon as one of the firms gets over this critical level. We begin by constructing a modified admissible strategy, which differs from the one presented in [1] on the definition of the critical level, and we use the Kuhn-Tucker conditions to prove its optimality. Different from [1], we change the Inada condition at 0 of the running revenue, allowing for a finite first derivative. It turns out that, also with these modifications, the strategy depicted in [1] is valid also in this case. Once we know the solution for the N firm problem, we study the asymptotic behavior of the problem as $N \to \infty$. By reformulating the N-firm problem using random measures, we are able to prove that the optimal solution for N players converge, as $N \to +\infty$, to a random measure which corresponds to a sudden installation which saturates the total capacity θ as soon as the electricity price reaches a certain threshold. For this result to hold, it is necessary that the marginal utility at 0 is finite, i.e. that we change the Inada condition at 0. Finally, we define the mean field control version of our problem and we prove existence of a solution by using an extension of Komlos’ theorem.

[1] M.B. Chiarolla, G. Ferrari, F. Riedel. Generalized Kuhn-Tucker Conditions for N-Firm Stochastic Irreversible Investment under Limited Resources, SIAM Journal on Control and Optimization 51(5) (2013), pp. 3863-3885.

Bagagiolo Fabio

(Università di Trento)

“On a single player vs a mass: a differential game and a related mean field (differential) game”

I consider a differential game where a single player and a whole mass control their own evolution in order to minimize and to maximize, respectively, a given cost functional. I derive the corresponding infinite dimensional Hamilton-Jacobi-Isaacs equations, and give a uniqueness result for them, on a suitable subset of a Hilbert space. Then I say something about a possible "mean field game" where a finite number of single agents respectively play a differential game versus the mass. Joint research program with Rossana Capuani and Luciano Marzufero.

Bonesini Ofelia

(Imperial College London)

“Correlated equilibria for mean field games with progressive strategies”

Abstract. In a discrete space and time framework, we study the mean field game limit for a class of symmetric N-player games based on the notion of correlated equilibrium. We give a definition of correlated

solution that allows to construct approximate N-player correlated equilibria that are robust with respect to progressive deviations. We illustrate our definition by way of an example with explicit solutions and we provide a direct proof for the existence of correlated solution for the MFG in restricted strategies.

Brambilla Chiara

(Università di Padova)

"The impact of tax benefits on reshoring: a Markov chain-based model"

Abstract. In recent years, tax benefits have been widely used to promote and support reshoring, which involves moving production from foreign countries back to the domestic market. Policymakers offer firms that engage in reshoring a significant taxes discount, aiming to balance the difference between the domestic and foreign production costs and cover the cost of repositioning the firms. In this paper, we present a model that describes the strategy of a large number of firms that have to consider the trade-offs between production costs, repositioning, and tax benefits. We use a continuous-time Markov chain with a binary state to describe the locations of production plants (domestic vs foreign) and employ mean-field games to model the problem, assuming complete symmetry among firms and a finite programming horizon. We study three different scenarios depending on the reduction policy: the trade-off between production and repositioning costs without any tax reductions, the effects of a uniform tax benefit on reshoring, and a mean-field game approach in which tax benefits are proportional to the firms that still produce abroad. Our main result is to assess the effectiveness of tax benefits in promoting the relocation of firms to domestic markets.

Callegaro Giorgia

(Università di Padova)

"Optimal reinsurance via BSDEs in a partially observable contagion model with jump clusters"

Abstract. We investigate the optimal reinsurance problem when the loss process exhibits jump clustering features and the insurance company has restricted information about the loss process. We maximize expected exponential utility of terminal wealth and show that an optimal solution exists. By exploiting both the Kushner-Stratonovich and Zakai approaches, we provide the equation governing the dynamics of the (infinite-dimensional) filter and characterize the solution of the stochastic optimization problem in terms of a BSDE, for which we prove existence and uniqueness of solution. After discussing the optimal strategy for a general reinsurance premium, we provide more explicit results in some relevant cases.

Cecchin Alekos

(Università di Padova)

"A mean field model for the development of renewable capacities"

Abstract. We propose a model based on a large number of small competitive producers of renewable energies, to study the effect of subsidies on the aggregate level of capacity, taking into account a cannibalization effect. We first derive a model to explain how long-time equilibrium can be reached on the market of production of renewable electricity and compare this equilibrium to the case of monopoly. Then we consider the case in which other capacities of production adjust to the production of renewable energies. The analysis is based on a master equation and we get explicit formulae for the long-time equilibria. Thus we find the optimal subsidies to be given by a central planner to the installation and the production in order to reach a desired equilibrium capacity. Based on joint work with Clémence Alasseur, Matteo Basei and Charles Bertucci.

Cirant Marco

(Università di Padova)

“Nash equilibria in large population stochastic games on sparse graphs”

Abstract. I will discuss the issue of describing and approximating Nash equilibria in a differential game involving many players whose interactions are described by a graph. I will consider in particular a framework where each player interacts with few close players only. Under some structural assumptions, I will show how to determine in a quantitative way the dependence of the equilibrium trajectories with respect to the states of the players. This will yield “localisation” results. Joint work with D. Redaelli (Padova).

Faggian Silvia

(Università Ca' Foscari Venezia)

“Growth Models with Externalities on Networks”

Abstract. This study examines the dynamics of capital stocks distributed among several nodes, representing different sites of production and connected via a weighted, directed network. The network represents the externalities or spillovers that the production in each node generates on the capital stock of other nodes. A regulator decides to designate some of the nodes for the production of a consumption good to maximise a cumulative utility from consumption. It is demonstrated how the optimal strategies and stocks depend on the productivity of the resource sites and the structure of the connections between the sites. The best locations to host production of the consumption good are identified per the model's parameters and correspond to the least central (in the sense of eigenvector centrality) nodes of a suitably redefined network that combines both flows between nodes and the nodes' productivity.

Ghilli Daria

(Università di Pavia)

“Linear quadratic Mean Field Games in infinite dimension and applications”

Abstract. We study a class of linear quadratic Mean Field Games (MFG) in infinite dimension, where the state variable lives in a Hilbert space. A MFG is a system of two partial differential equations describing Nash equilibria in large populations of agents. In the class of problems we study the state equation is a PDE or a delay equation which can be written as an ODE in a suitable Hilbert space. As a first step we consider the case when the objective of the agents depends on the mean of their state. A typical example of this situation is when the aim of the agents is to minimize the deviation from the mean. We show how the problem can be reduced to a system of a Riccati equation and two coupled ODEs in infinite dimension and we solve the latter by disentagling the system and solving an additional Riccati equation in infinite dimension. Moreover we provide some applications to a vintage capital model where the state equation for the capital is a first order PDE and to a model of inter-bank borrowing and lending where the state equation (dynamics of the monetary reserves of the banks) is a stochastic delayed differential equation.

Lanaro Giacomo

(Università di Padova)

“Price formation under asymmetry of information via mean-field game techniques“

Abstract. In financial markets, quantifying the information possessed by an agent trading an asset is a crucial task, especially when there is no homogeneity between the amount of information that can be

accessed by every player. Our purpose is to study the behaviour of an equilibrium price determined by the market clearing condition (i.e. the match between the demand and the supply) between rational financial agents who observe different amounts of information. We focus on a market in which one asset is traded by N less informed agents and one major agent. We show that, under the observation of the equilibrium price process, the less informed players bridge the gap in terms of the amount of available information. However, the equation for the price process in the game with finitely many players is not tractable, thus we prove the existence of a mean-field solution to the equation for the price process when $N \to \infty$. To do so, we apply techniques related to the existence of weak mean-field game equilibria. We show that the construction of the price process in the mean-field limit guarantees a weak form of the market clearing condition.

Maggistro Rosario

(Università di Trieste)

“A dynamic game approach for optimal consumption, investment and life insurance problem”

Abstract. We consider a multi-agent portfolio optimization model, with life insurance, for two players with random lifetime under a dynamic game approach. Each player is a price-taker and invests in the market in order to maximize her own utility for consumption and bequest. The market is complete and consists of n different assets, of which $n-1$ are risky with prices driven by Geometric Brownian motion, while one is risk-free. We analyse both the non-cooperative and cooperative scenarios, and by considering the family of CRRA utilities functions we determine the closed-form expressions of the optimal consumption, investment, and life insurance for both players. A sensitivity analysis is provided both to illustrate the impact of the biometric and risk aversion parameters on the optimal controls and to compare the non-cooperative strategies with the cooperative ones.

Marzufero Luciano

(Università La Sapienza Roma)

“A Mean-Field Game network model for urban planning”

Abstract. We study a mathematical model to describe the evolution of a city, which is determined by the interaction of two large populations of agents, workers and firms. The map of the city is represented by a network with the edges representing at the same time residential areas and communication routes. We obtain a two population Mean-Field Game system coupled with an Optimal Transport problem defined on the network. We prove existence and uniqueness of the solution and several numerical simulations are also provided.

Muttoni Maddalena

(Università di Padova)

“Optimal control in anticipation of a regime shift: a co-state analysis”

Abstract. An innovative approach to two-stage optimal control with a stochastic switching time consists in reformulating the problem as a deterministic, vintage-structured optimal control problem. A suitable version of the Maximum Principle then provides necessary conditions for optimal solutions, in the form of maximum conditions and co-state equations for both stages. We analyze the co-state equations in the anticipative stage: by comparing them to the equations that arise if a switch is not expected, we identify the role played by anticipation in shaping the optimal strategy prior to the regime shift.

Tonon Daniela

(Università di Padova)

"Mean Field Games planning problems with general initial and final measures"

Abstract. The planning problem in Mean Field Games (MFG) was introduced by P.-L. Lions in his lessons, to describe models in which a central planner would like to steer a population to a predetermined final configuration while still allowing individuals to choose their own strategies. In a recent variational approach, see Graber, Mészáros, Silva and Tonon (2019) and Orrieri, Porretta and Savaré (2019) the authors studied the well-posedness of this problem in case of merely summable initial and final measures, using techniques, coming from optimal transport, introduced by Benamou and Brenier in 2000, extended to the congestion case in Carlier, Cardaliaguet and Nazaret (2013), and already used to show the existence and uniqueness of weak solutions for classical MFGs by Cardaliaguet and collaborators. The case of less regular initial and final measures is now studied in collaboration with F.J. Silva via techniques introduced by Jimenez in 2008, for the analogous problem in optimal transport.