Necessary higher-order optimality conditions involving Lie bracket for impulsive optimal control problems

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We consider control systems governed by nonlinear ordinary differential equations of the form:

$$\dot{x}(t) = f(x(t), u(t), v(t)) + \sum_{\alpha=1}^{m} g_\alpha(x(t), u(t))\dot{u}_\alpha(t), \quad for\ t \in [0, T],$$

where $x : [0, T] \to R^n$ is the *state variable,* $u : [0, T] \to R^m$ is the *impulsive control* and $v : [0, T] \to R^l$ is the *ordinary control.* The control u is allowed to be a (discontinuous) bounded variation function, which gives the system an impulsive character. For this class of systems, we adopt the concept of raph completion solution that was introduced by A. Bressan and F. Rampazzo in the 90's. We consider an optimal control problem in the Mayer form, with general control and final state constraints, for which we prove a maximum principle and higher-order necessary conditions in terms of the adjoint state and the Lie brackets of the involved vector fields.