Lack of BV bounds for impulsive control systems

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Abstract

In this talk I will present some results obtained in the last few years in collaboration with Monica Motta [2], [3]. For a nonlinear impulsive control system

$$\begin{array}{l} \dot{x}(t) = g_0(x(t), u(t), v(t)) + \sum_{i=1}^{m} g_i(x(t), u(t))\dot{u}_i(t), \quad t \in]0, T], \\ x(0) = \bar{x}, \quad u(0) = \bar{u}, \end{array} \tag{1}$$

where $x \in \mathbb{R}^n$ and the measurable control pair (u, v) ranges over a compact set $U \times V \subset \mathbb{R}^m \times R^l$, we extend the so-called graph completion approach and introduce a notion of generalized solution x associated to a control u whose total variation is bounded on $[0, t]$ for every $t < T$, but possibly unbounded on $[0, T]$. We prove existence, consistency with classical solutions and well-posedness of this solution. In particular, we characterize it as a *simple limit solution*, that is the pointwise limit of certain regular solutions, extending a result due to Aronna and Rampazzo [1], for the case of bounded variation. Moreover starting with an example of optimal control where the minimum does not exist in the class of limit solutions, we propose a notion of *extended limit solution* x, for which such a minimum exists. As a first result, we prove that extended BV (respectively, BV_{loc}) simple limit solutions and BV (respectively, BV_{loc}) simple limit solutions coincide. Finally we address a problem that was left open in [1], that is the definition of limit solutions for dynamics where the ordinary control v also appears in the non-drift terms g_i, $i = 1, \dots, m$. For the associated system we prove that, in the BV case, the notion of *extended limit solutions* coincide with that of graph completion solutions. The notion that we consider provides the natural setting for controllability questions and for some non-coercive optimal control problems, where chattering phenomena at the final time are expected. More in general, it is well suited to describe the evolution of control systems subject to a train of impulses where no a-priori bounds on the number and the amplitude of the impulses are imposed.

[1] S. Aronna and F. Rampazzo, L^1 *limit solutions for control systems*, J. Differential Equations 258 (2015), no. 3, 954979
[2] M. Motta and C. S., *Lack of BV bounds for impulsive control systems*, JMAA 461 (2018), no. 1, 422–450.
[3] M. Motta and C. S., *On* L^1 *limit solutions in impulsive control*, to appear on DCDS-S.