Lack of BV bounds for impulsive control systems

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Abstract

In this talk I will present some results obtained in the last few years in collaboration with Monica Motta [2], [3]. For a nonlinear impulsive control system

\[
\dot{x}(t) = g_0(x(t), u(t), v(t)) + \sum_{i=1}^{m} g_i(x(t), u(t))\dot{u}_i(t), \quad t \in [0, T],
\]

where \( x \in \mathbb{R}^n \) and the measurable control pair \((u, v)\) ranges over a compact set \( U \times V \subset \mathbb{R}^m \times \mathbb{R}^l \), we extend the so-called graph completion approach and introduce a notion of generalized solution \( x \) associated to a control \( u \) whose total variation is bounded on \([0, t]\) for every \( t < T \), but possibly unbounded on \([0, T]\). We prove existence, consistency with classical solutions and well-posedness of this solution. In particular, we characterize it as a simple limit solution, that is the pointwise limit of certain regular solutions, extending a result due to Aronna and Rampazzo [1], for the case of bounded variation. Moreover starting with an example of optimal control where the minimum does not exist in the class of limit solutions, we propose a notion of extended limit solution \( x \), for which such a minimum exists. As a first result, we prove that extended BV (respectively, BV_{loc}) simple limit solutions and BV (respectively, BV_{loc}) simple limit solutions coincide. Finally we address a problem that was left open in [1], that is the definition of limit solutions for dynamics where the ordinary control \( v \) also appears in the non-drift terms \( g_i, i = 1, \ldots, m \). For the associated system we prove that, in the BV case, the notion of extended limit solutions coincide with that of graph completion solutions. The notion that we consider provides the natural setting for controllability questions and for some non-coercive optimal control problems, where chattering phenomena at the final time are expected. More in general, it is well suited to describe the evolution of control systems subject to a train of impulses where no a-priori bounds on the number and the amplitude of the impulses are imposed.