Shape optimization problems on doubly connected domains

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The problem of finding a domain, in a given class of domains, which optimizes the first nonzero eigenvalue of certain eigenvalue problem has been extensively studied in the literature. In this talk, we discuss about the following similar problems:

1. For $n > 2$, let B_1 and B_2 be two open balls in $\mathbb{R}^n$ of fixed radius such that $\overline{B}_1 \subset B_2$. Consider the eigenvalue problem
$$\begin{array}{rcll} \Delta\varphi &=& 0 & \text{in } B_2 \setminus \overline{B}_1, \\ \varphi &=& 0 & \text{on } \partial B_1, \\ \frac{\partial\varphi}{\partial\nu} &=& \tau\varphi & \text{on } \partial B_2, \end{array} \tag{1}$$
where ν is the outward unit normal to ∂B_2. Then the first eigenvalue of (**??**) attains its maximum if and only if B_1 and B_2 are concentric.

2. Let $(\mathbb{M}, ds^2)$ be a non-compact rank-1 symmetric space. Let $B_0 \subset M$ be a geodesic ball centered at a point $p \in M$, and $D \subset M$ be a domain of fixed volume such that $D = \exp_p(N_0)$, where N_0 is a symmetric neighborhood of the origin in T_pM and $\overline{B}_0 \subset D$. Consider the following problem
$$\begin{array}{rcll} \Delta\varphi &=& \mu\varphi & \text{in } D \setminus \overline{B}_0, \\ \frac{\partial\varphi}{\partial\nu} &=& 0 & \text{on } \partial\left(D \setminus \overline{B}_0\right), \end{array} \tag{2}$$
where ν is the outward unit normal to $\partial\left(D \setminus \overline{B}_0\right)$. Then the first nonzero eigenvalue of (**??**) attains its maximum if and only if D is a geodesic ball centered at p.

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