The problem of finding a domain, in a given class of domains, which optimizes the first nonzero eigenvalue of certain eigenvalue problem has been extensively studied in the literature. In this talk, we discuss about the following similar problems:

1. For $n > 2$, let $B_1$ and $B_2$ be two open balls in $\mathbb{R}^n$ of fixed radius such that $B_1 \subset B_2$. Consider the eigenvalue problem

$$\begin{align*}
\Delta \varphi &= 0 & \text{in } B_2 \setminus B_1, \\
\varphi &= 0 & \text{on } \partial B_1, \\
\frac{\partial \varphi}{\partial \nu} &= \tau \varphi & \text{on } \partial B_2,
\end{align*}$$

(1)

where $\nu$ is the outward unit normal to $\partial B_2$. Then the first eigenvalue of (1) attains its maximum if and only if $B_1$ and $B_2$ are concentric.

2. Let $(M, ds^2)$ be a non-compact rank-1 symmetric space. Let $B_0 \subset M$ be a geodesic ball centered at a point $p \in M$, and $D \subset M$ be a domain of fixed volume such that $D = \exp_p(N_0)$, where $N_0$ is a symmetric neighborhood of the origin in $T_p M$ and $B_0 \subset D$. Consider the following problem

$$\begin{align*}
\Delta \varphi &= \mu \varphi & \text{in } D \setminus B_0, \\
\frac{\partial \varphi}{\partial \nu} &= 0 & \text{on } \partial (D \setminus B_0),
\end{align*}$$

(2)

where $\nu$ is the outward unit normal to $\partial (D \setminus B_0)$. Then the first nonzero eigenvalue of (2) attains its maximum if and only if $D$ is a geodesic ball centered at $p$.

Based on joint work with Prof. G. Santhanam.

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